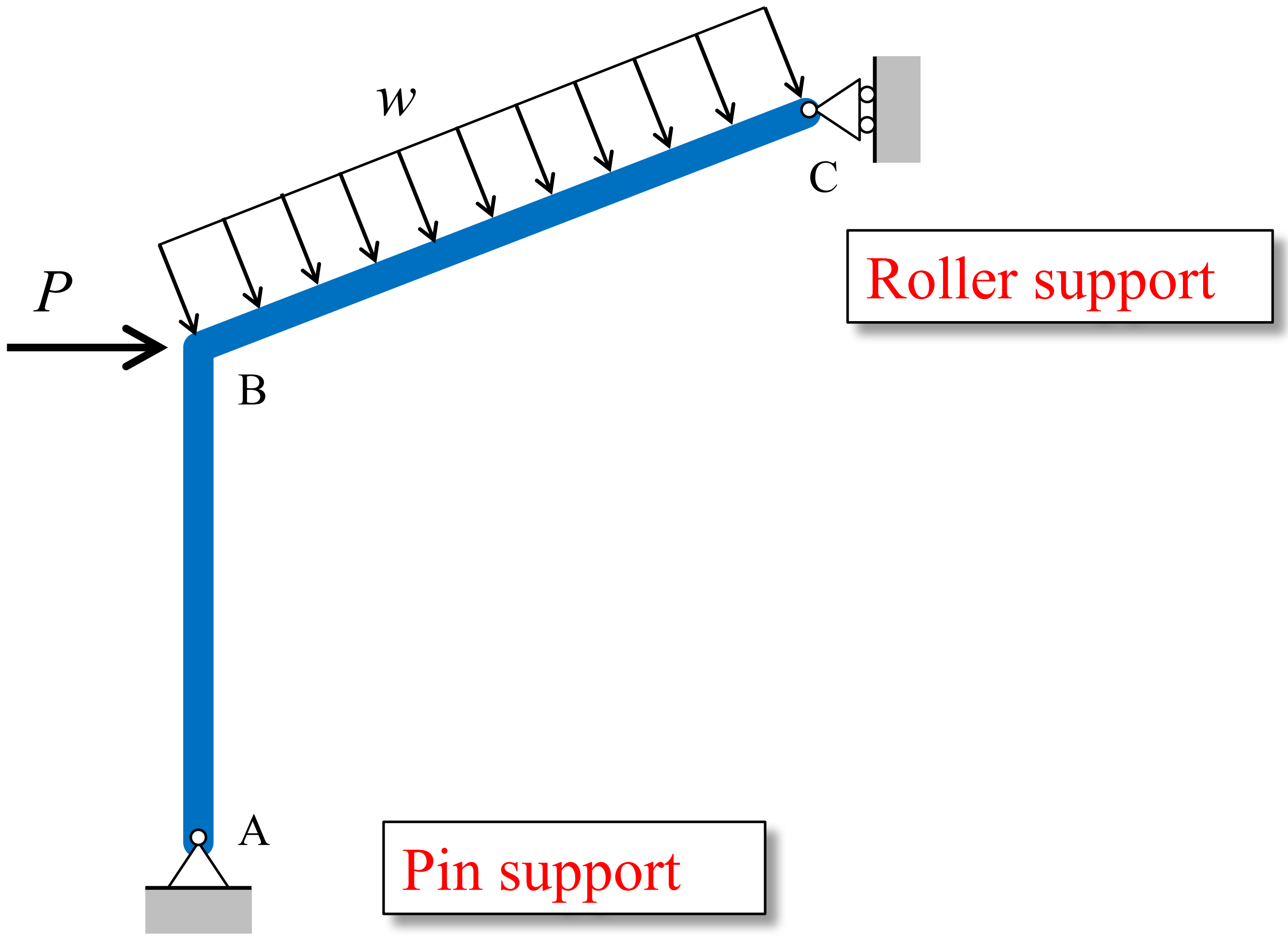
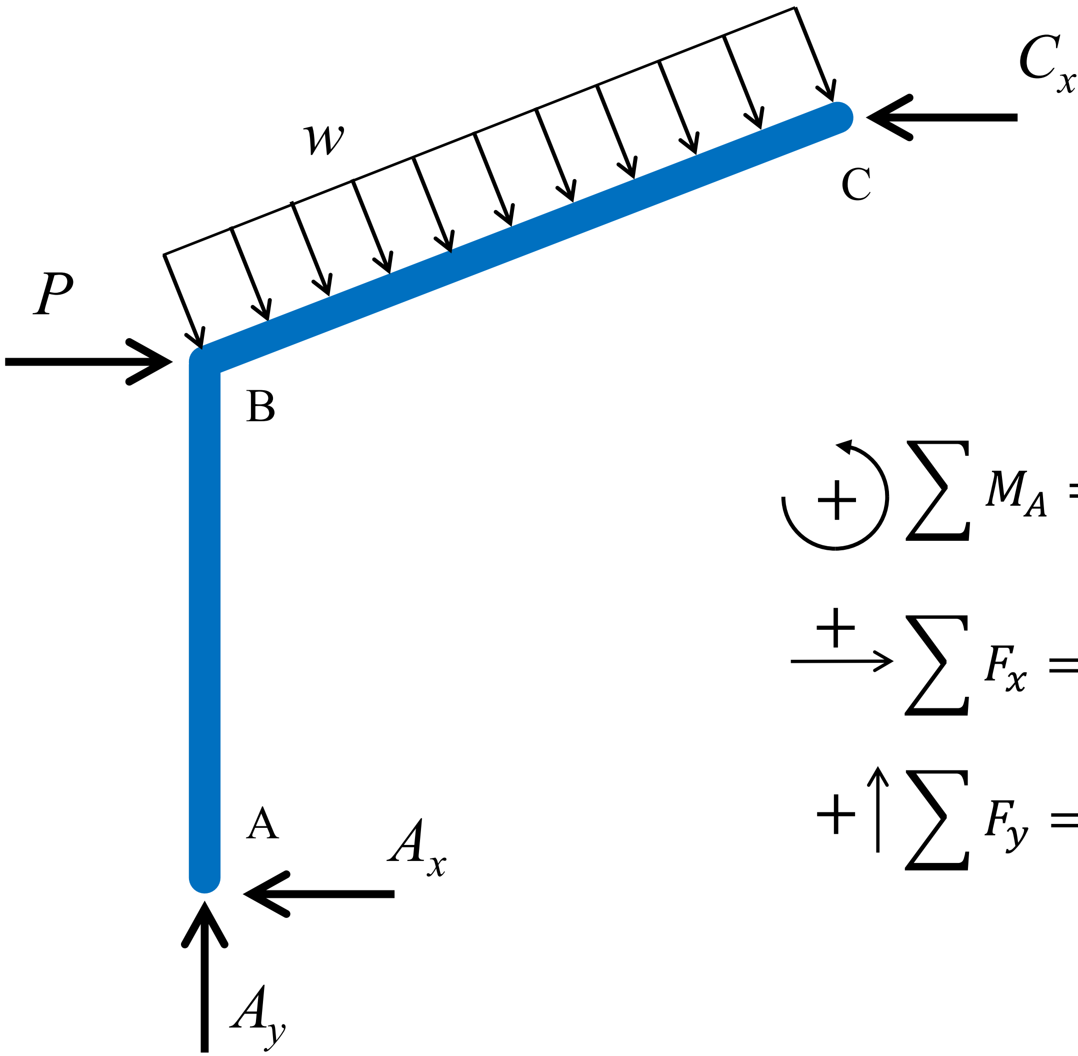
Internal Forces in Planar Structures

Steven Vukazich
San Jose State University

Consider the Frame



From an Equilibrium Analysis,
We Can Find the Support Reactions

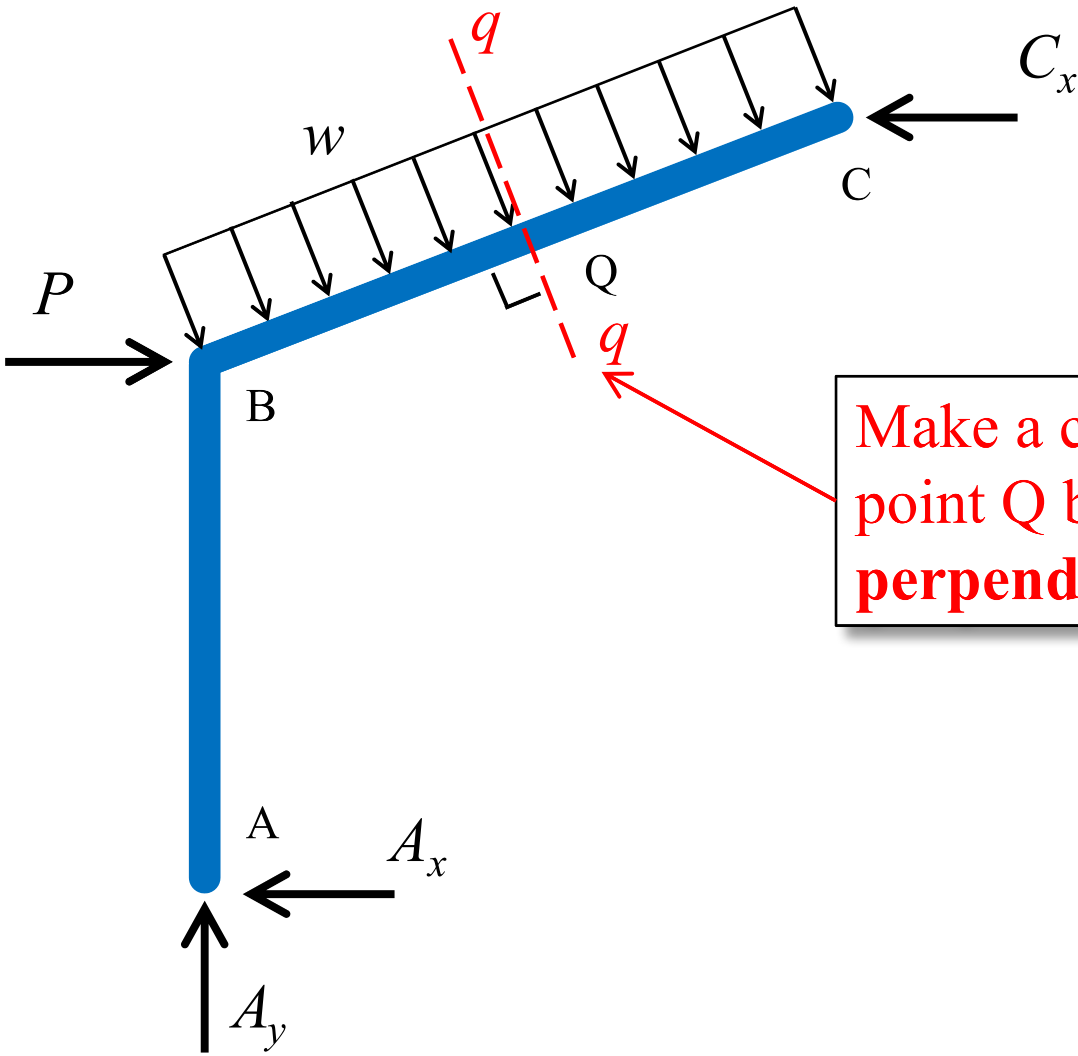


$$\circlearrowleft + \sum M_A = 0 \rightarrow C_x$$

$$\overset{+}{\rightarrow} \sum F_x = 0 \rightarrow A_x$$

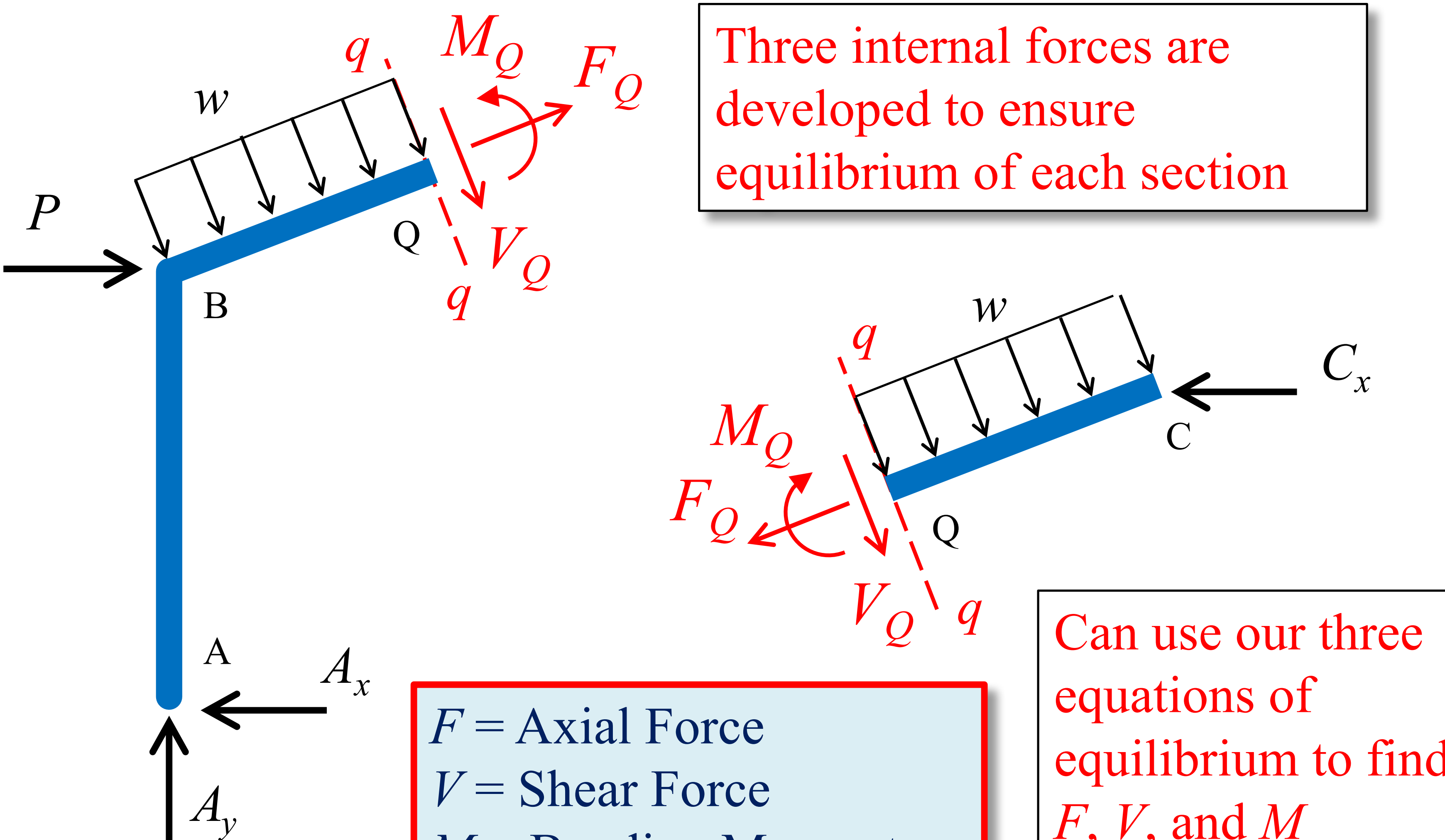
$$+\uparrow \sum F_y = 0 \rightarrow A_y$$

Cut the Frame into Two Sections



Make a cut at an arbitrary point Q between B and C **perpendicular** to member BC

Free Body Diagrams of the Sections of the Frame

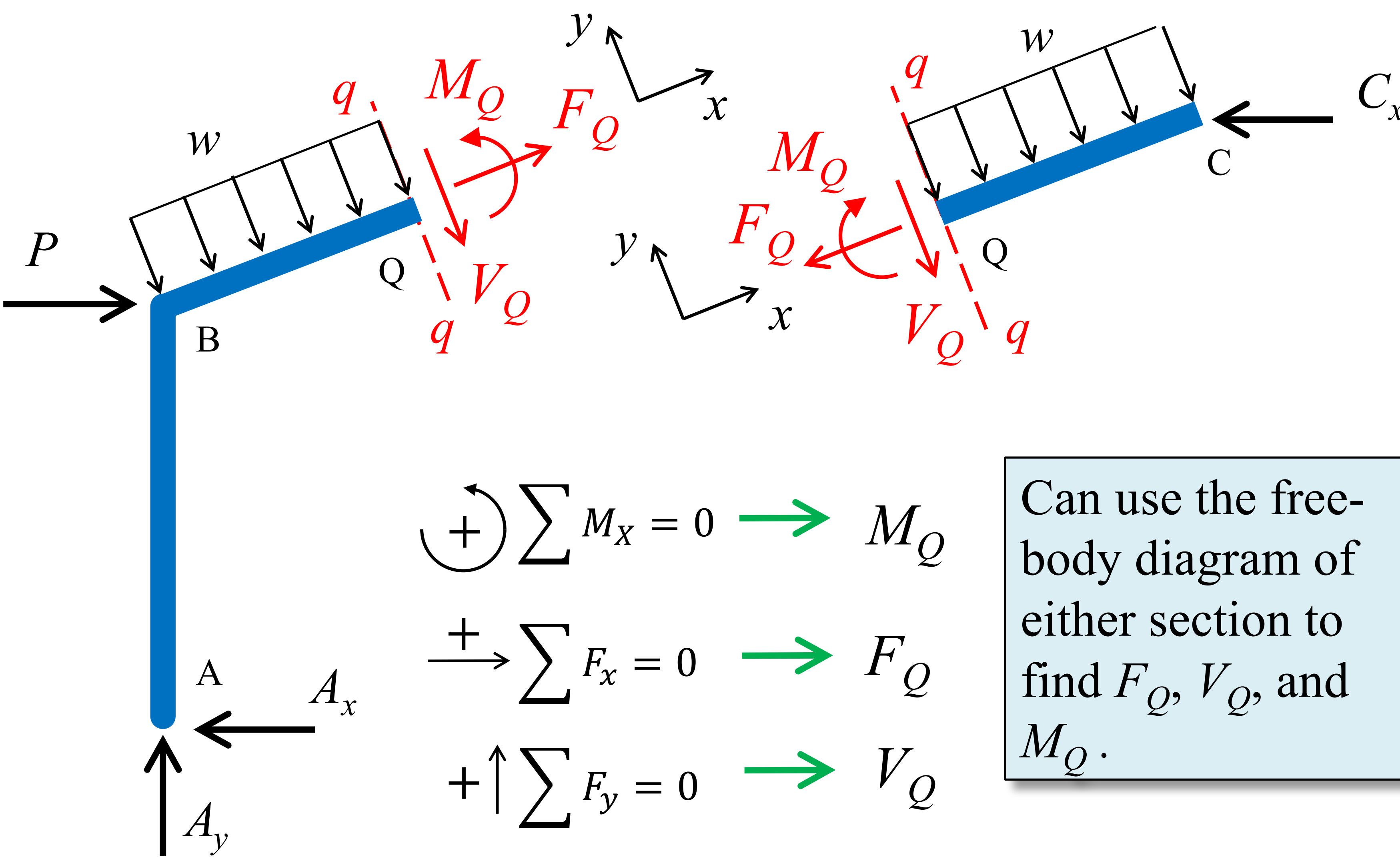


Three internal forces are developed to ensure equilibrium of each section

F = Axial Force
V = Shear Force
M = Bending Moment

Can use our three equations of equilibrium to find F, V, and M

Use Equilibrium to Find Internal Forces



$$\circlearrowleft + \sum M_x = 0 \rightarrow M_Q$$

$$\xrightarrow{+} \sum F_x = 0 \rightarrow F_Q$$

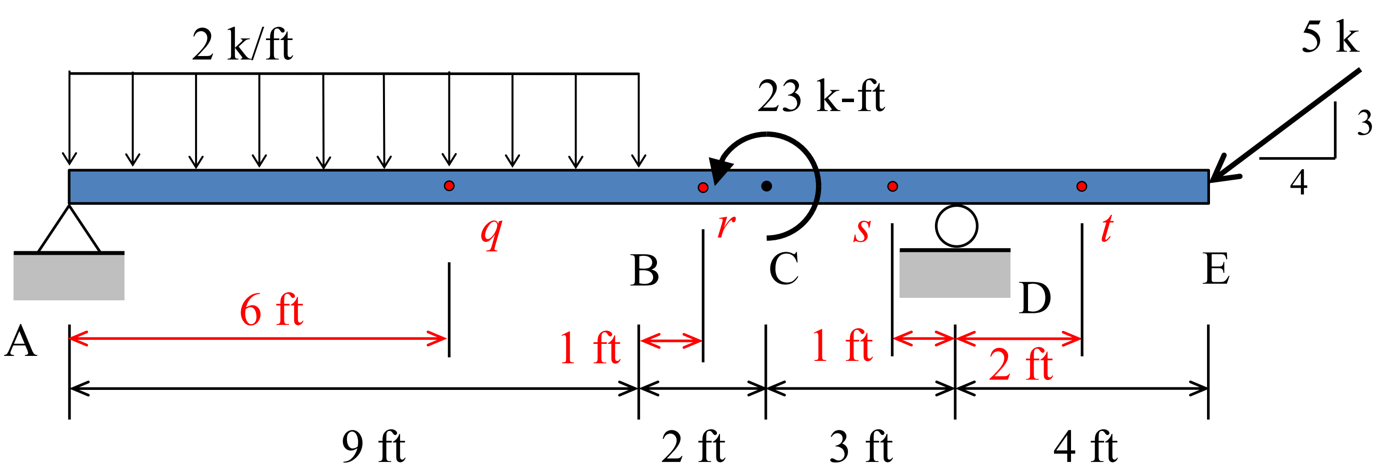
$$+\uparrow \sum F_y = 0 \rightarrow V_Q$$

Can use the free-body diagram of either section to find F_Q, V_Q, and M_Q.

Internal Force Example Problem

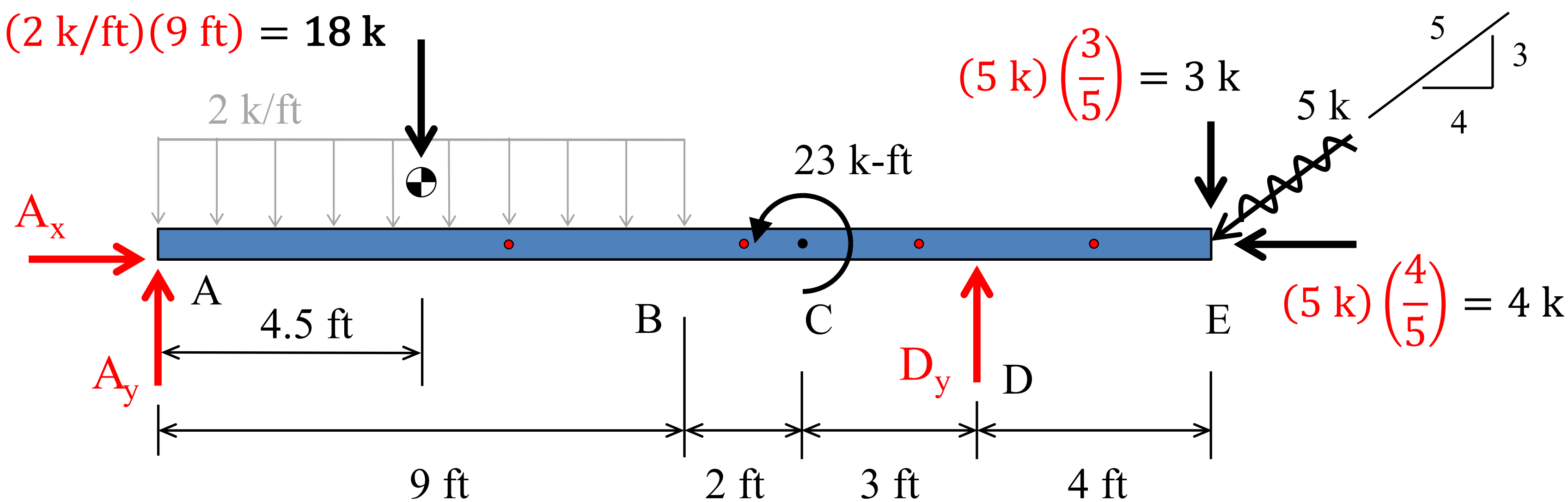
A beam is supported by a pin support at point A and extends over a roller support at point D. The beam is and subjected to a uniformly distributed load from A to B, a point moment at point C an inclined point load at point E as shown.

Find the internal forces at points *q*, *r*, *s*, and *t*.



Find All of the External Forces

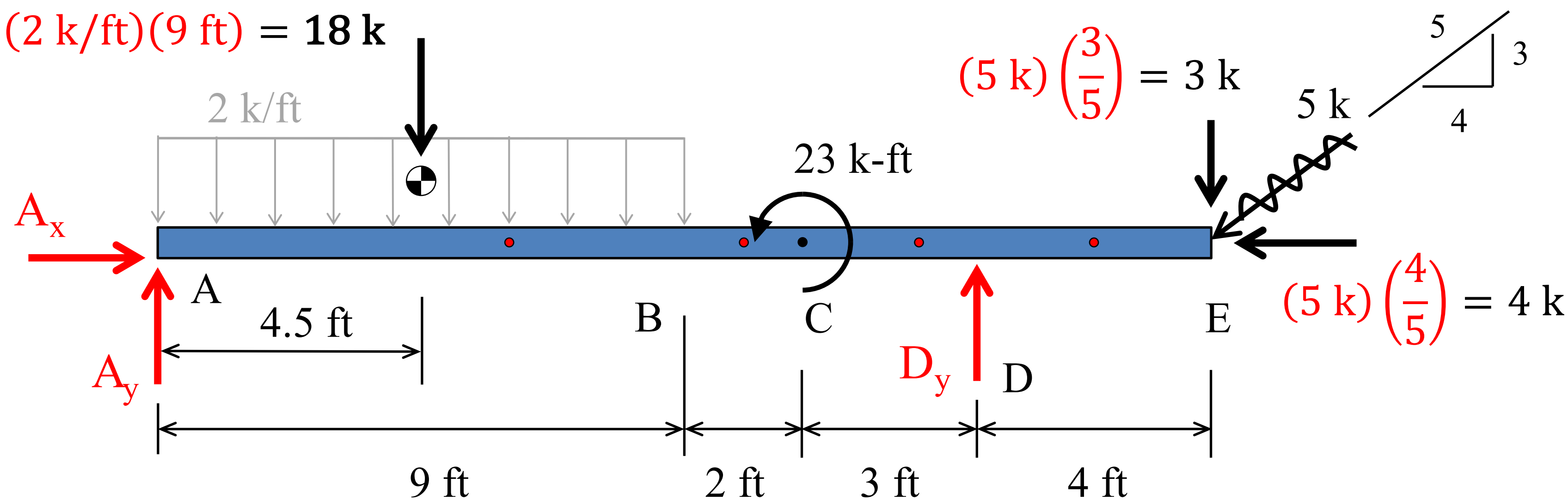
FBD of beam



$$+\circlearrowleft \sum M_A = 0$$

$D_y = 10$ k

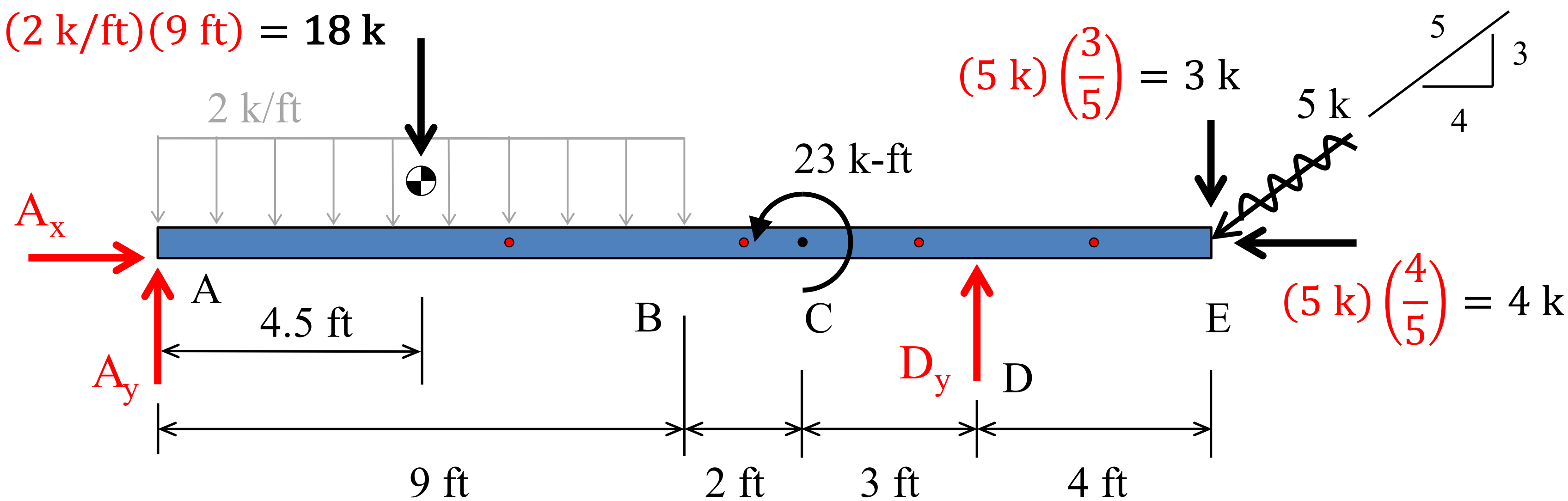
Find All of the External Forces



$$+\uparrow \sum F_y = 0$$

$A_y = 13$ k

Find All of the External Forces

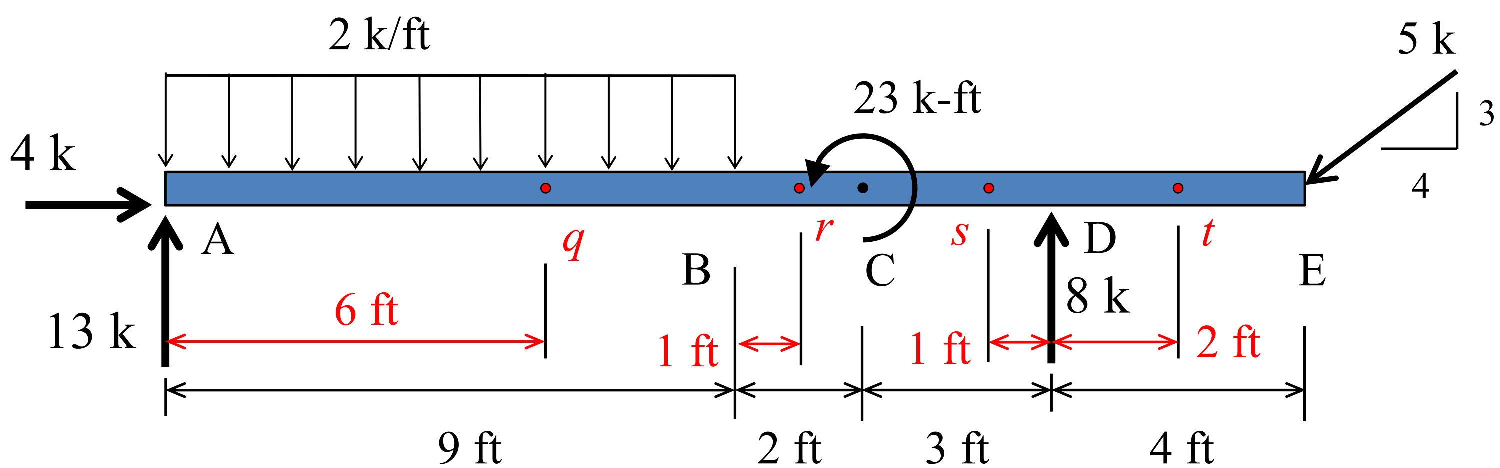


$$\xrightarrow{+} \sum F_x = 0$$

$A_x = 4$ k

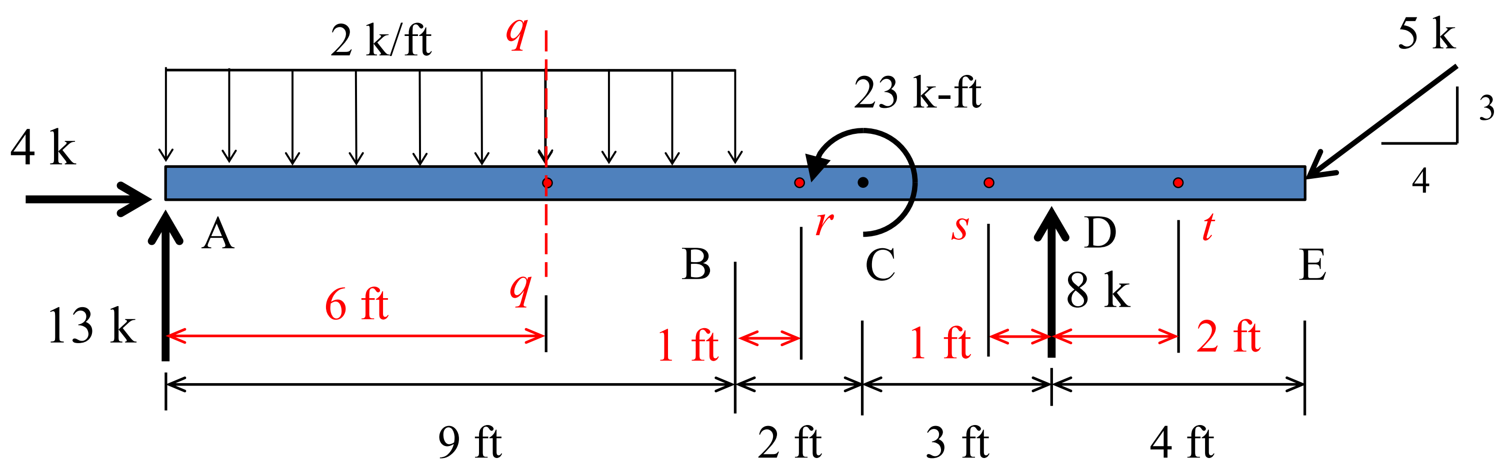
External Forces

FBD of beam showing all external forces



Find Internal Forces at Point *q*

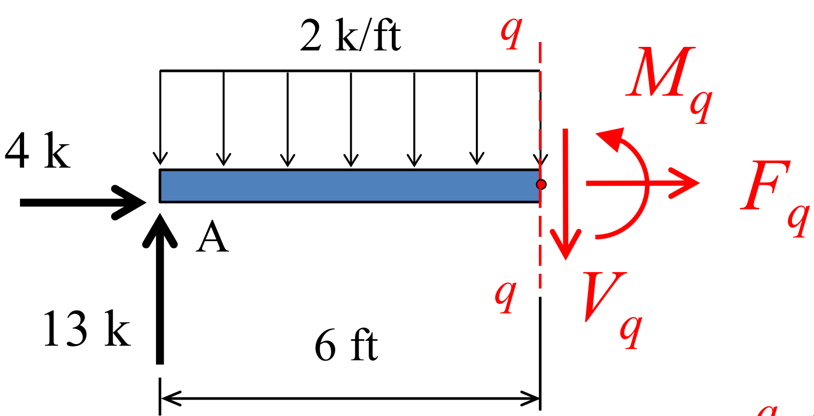
Cut beam at point *q*

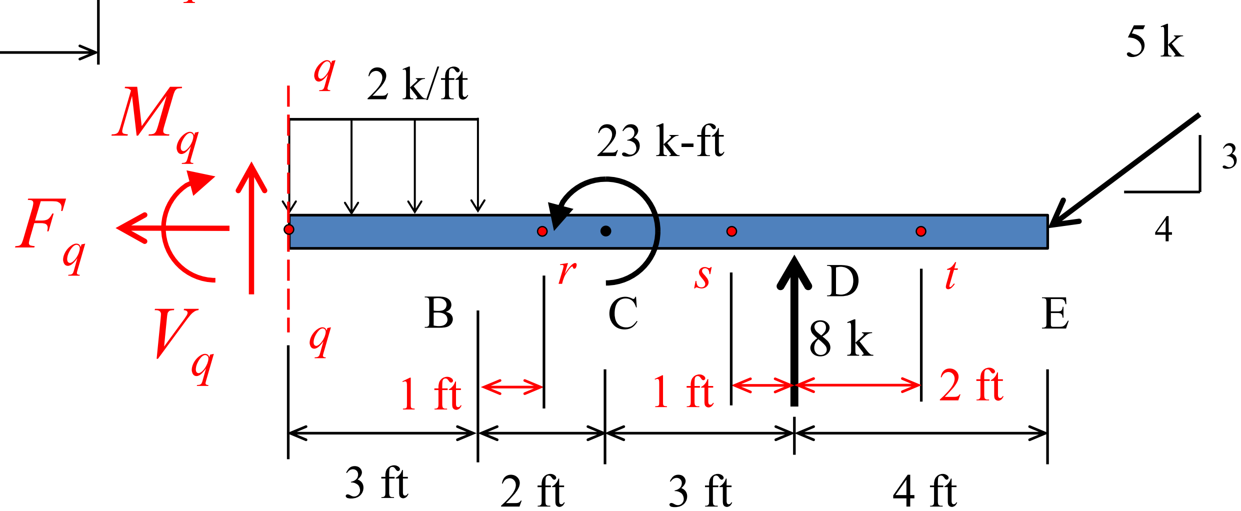


Find Internal Forces at Point q

FBDs of Segments Aq and qBCDE

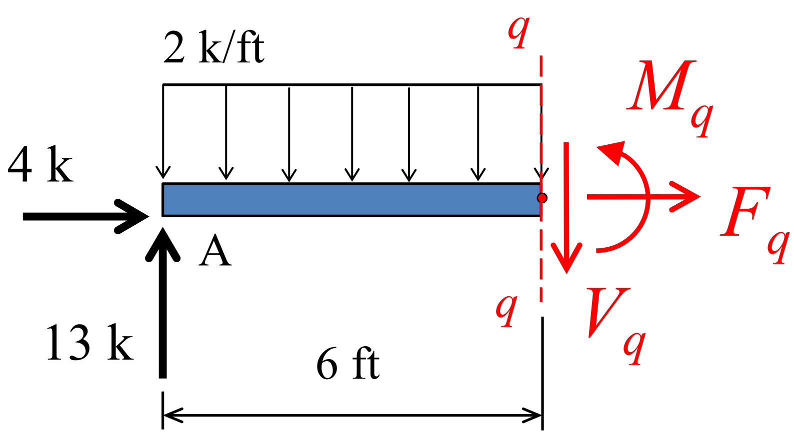
Can apply equations of equilibrium to either FBD to find internal forces

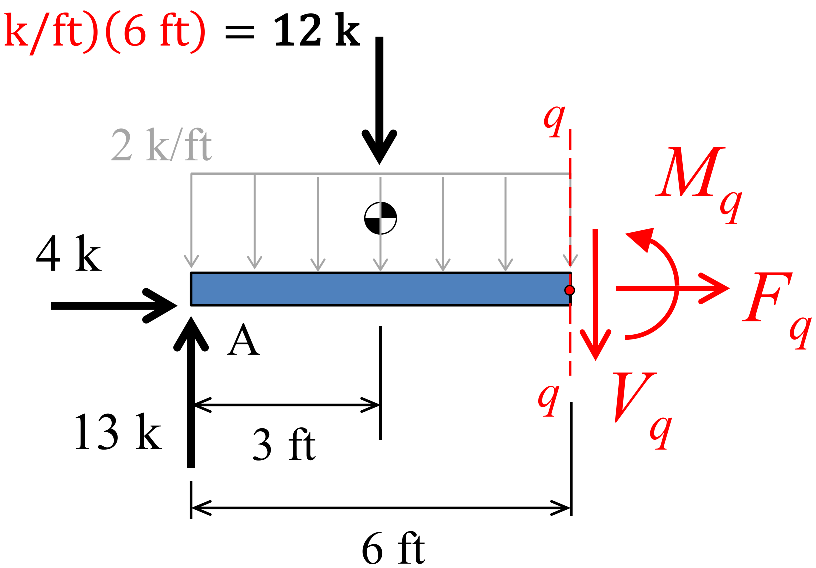




Find Internal Forces at Point q

FBD of Segments Aq





$$\circlearrowleft + \sum M_q = 0$$

$$+\uparrow \sum F_y = 0$$

$$\xrightarrow{+} \sum F_x = 0$$

$M_q = 42$ k-ft

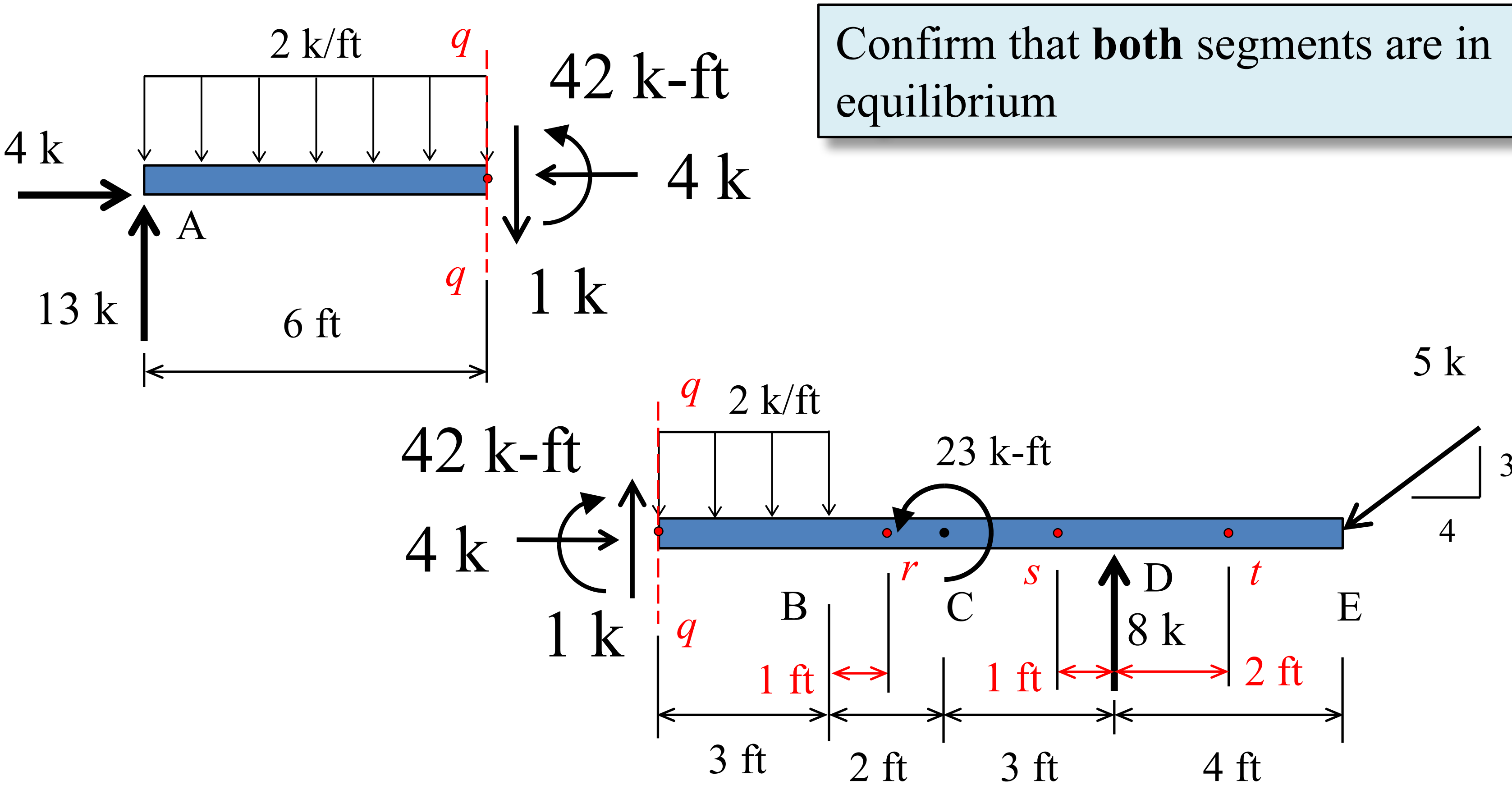
$V_q = 1$ k

$F_q = -4$ k

Internal Forces at Point *q*

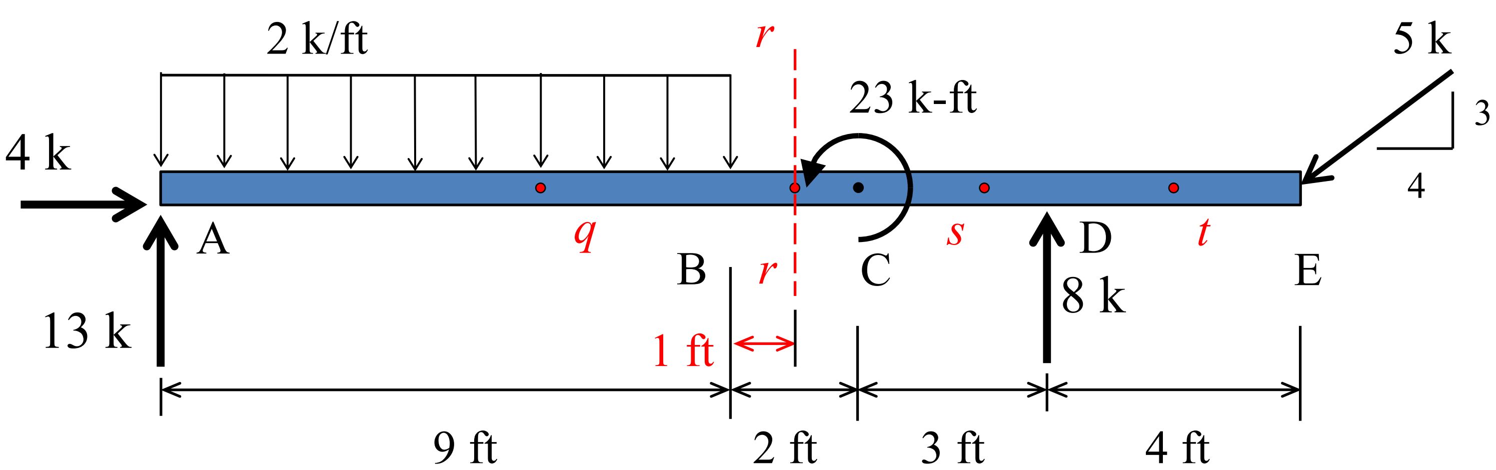
FBDs of Segments A*q* and *q*BCDE

Confirm that **both** segments are in equilibrium



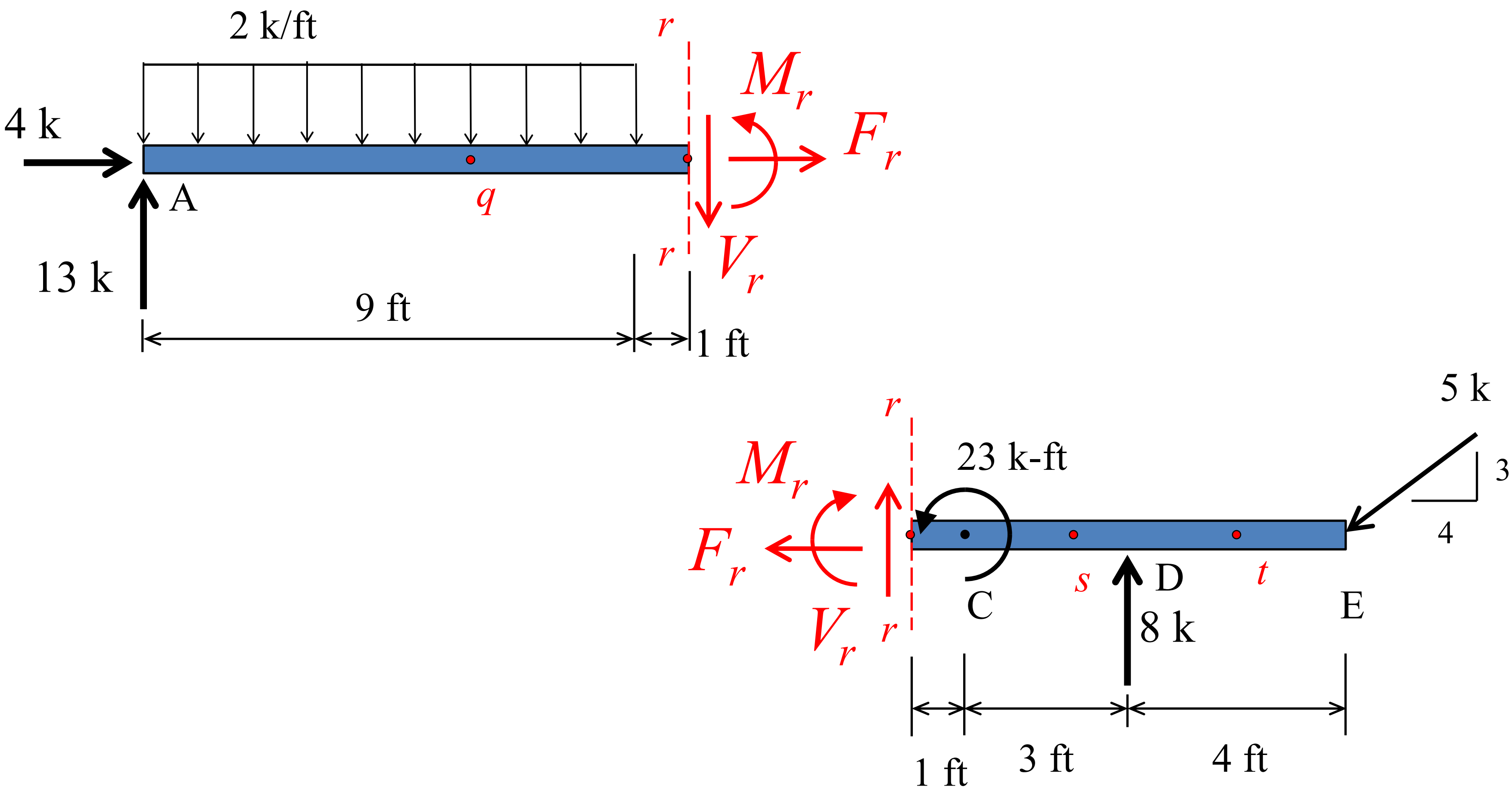
Find Internal Forces at Point *r*

Cut beam at point *r*



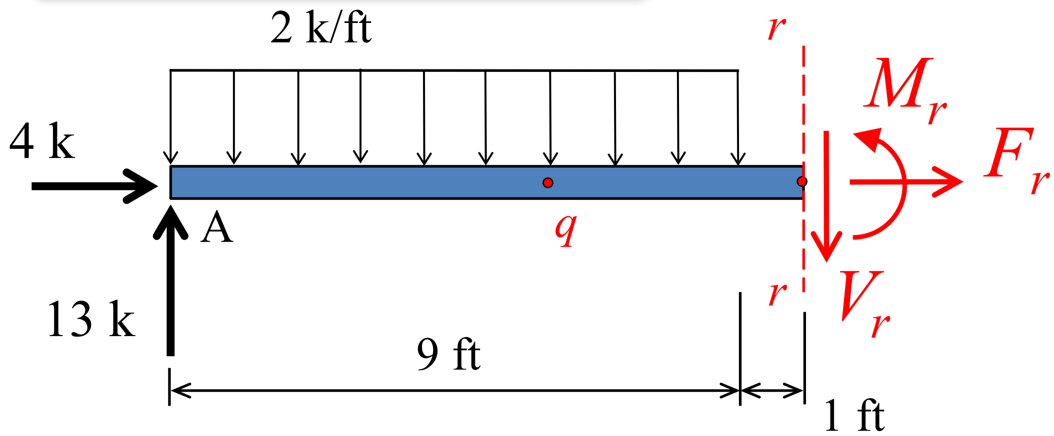
Find Internal Forces at Point *r*

FBDs of Segments AB*r* and *r*CDE



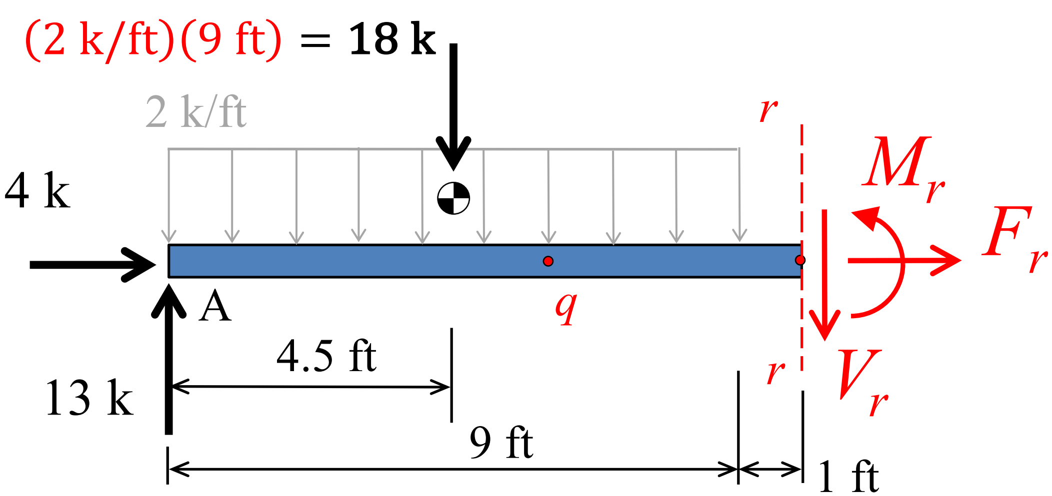
Find Internal Forces at Point r

FBD of Segment ABr



$$\circlearrowleft + \sum M_r = 0$$

$$+\uparrow \sum F_y = 0$$



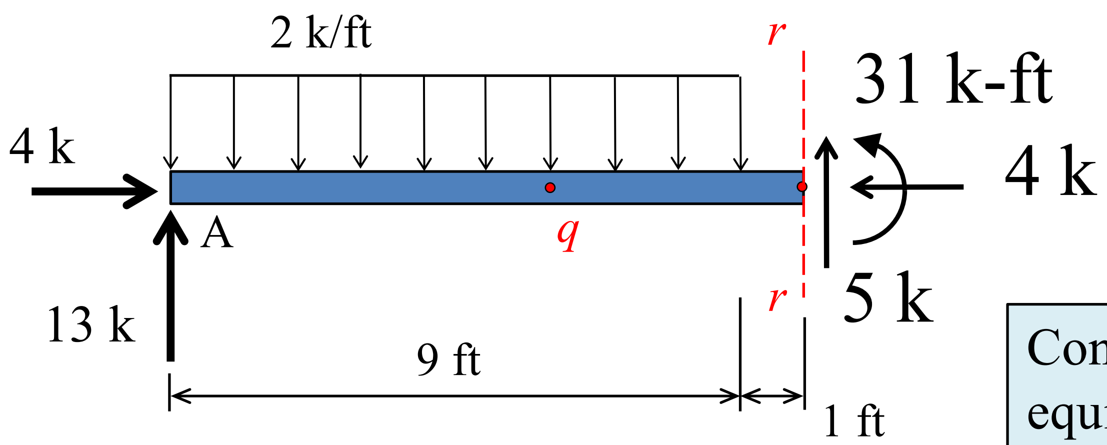
$$\xrightarrow{+} \sum F_x = 0$$

$M_r = 31$ k-ft

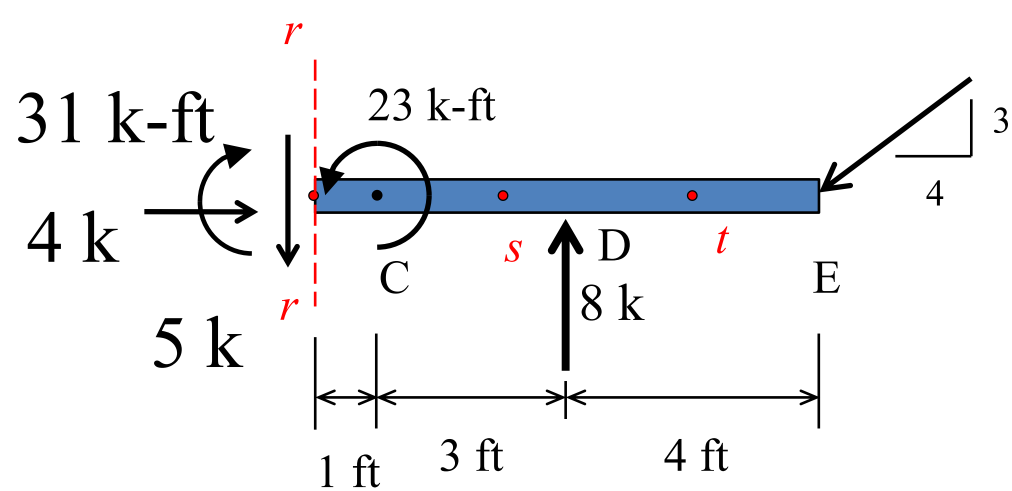
$V_r = -5$ k

$F_r = -4$ k

Internal Forces at Point *r*

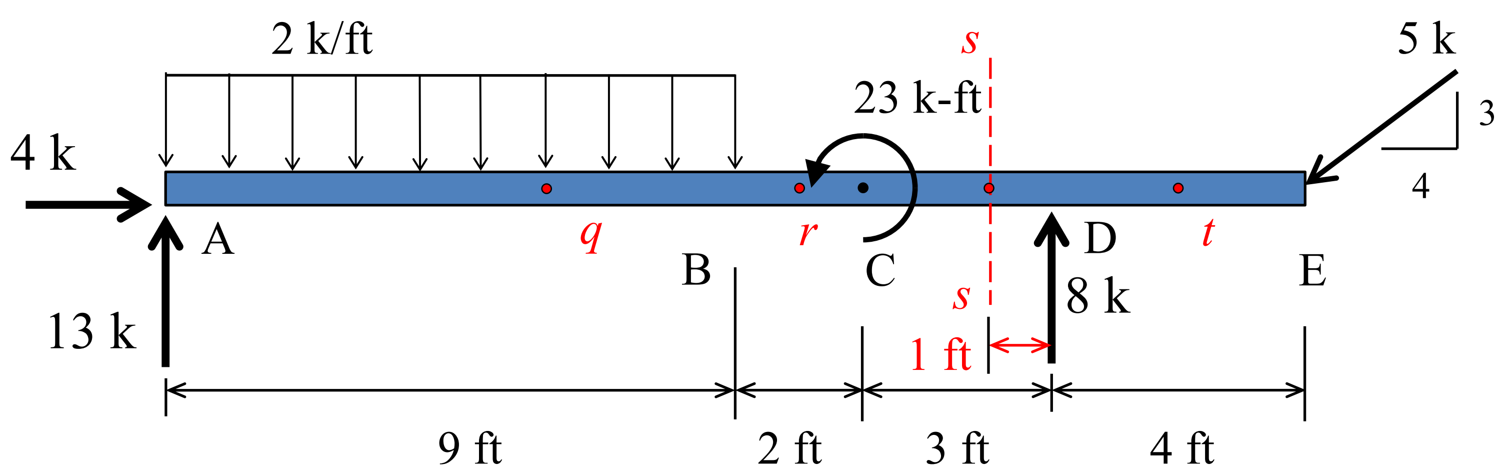


Confirm that **both** segments are in equilibrium



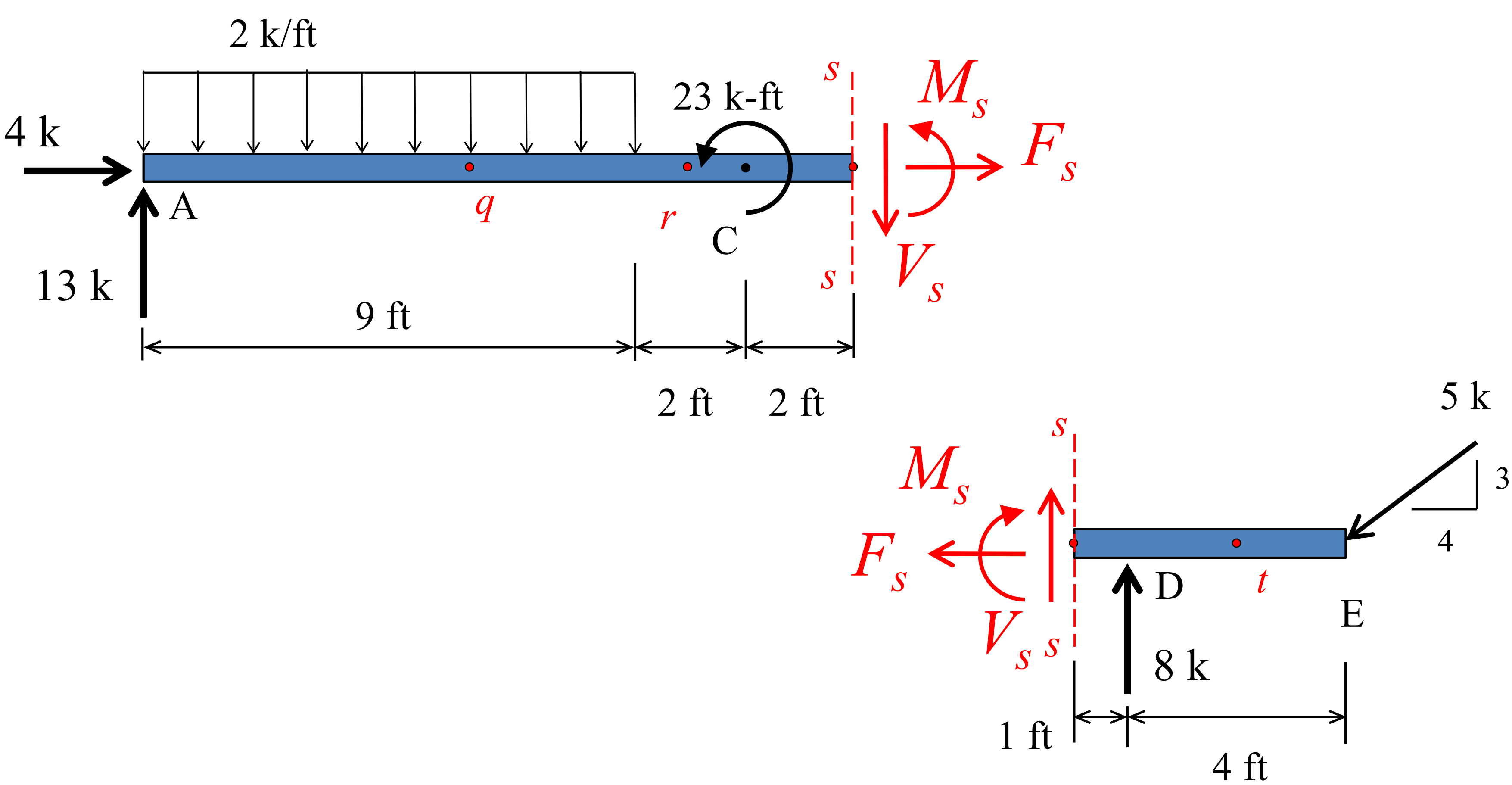
Find Internal Forces at Point *s*

Cut beam at point *s*



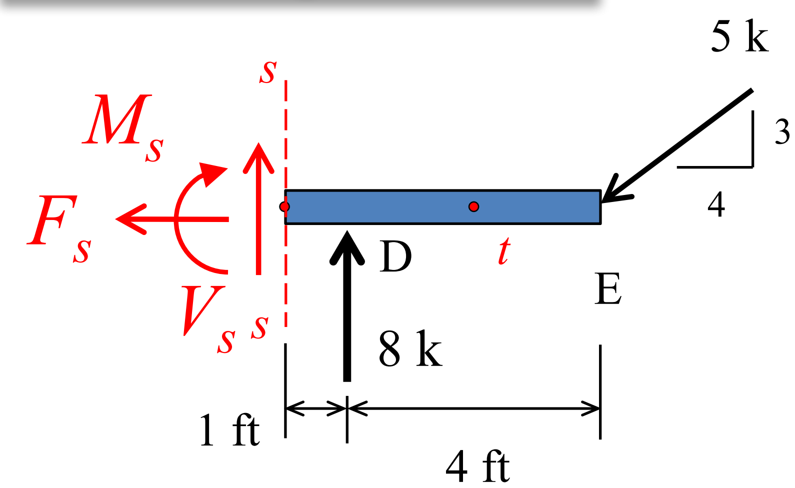
Find Internal Forces at Point *s*

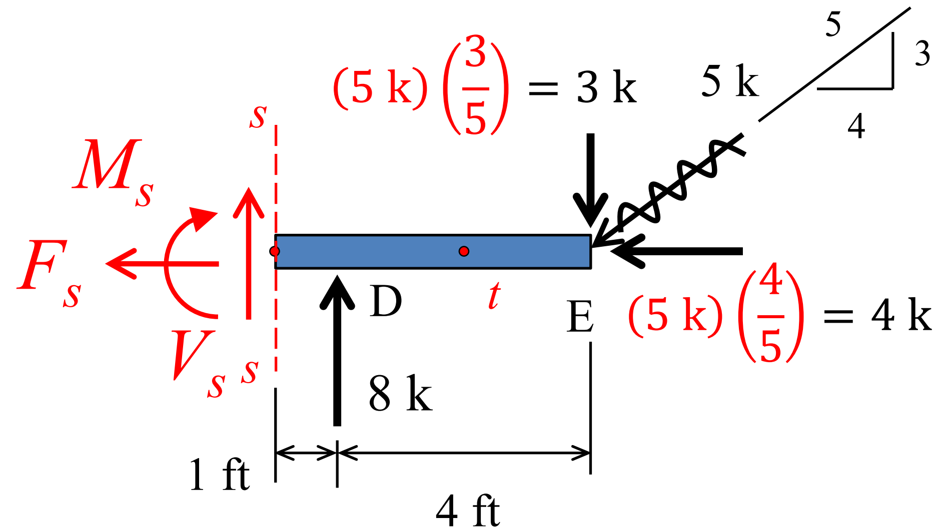
FBDs of Segments ABC*s* and *s*DE



Find Internal Forces at Point s

FBD of Segment sDE





$$\circlearrowleft + \sum M_s = 0$$

$$+\uparrow \sum F_y = 0$$

$$\xrightarrow{+} \sum F_x = 0$$

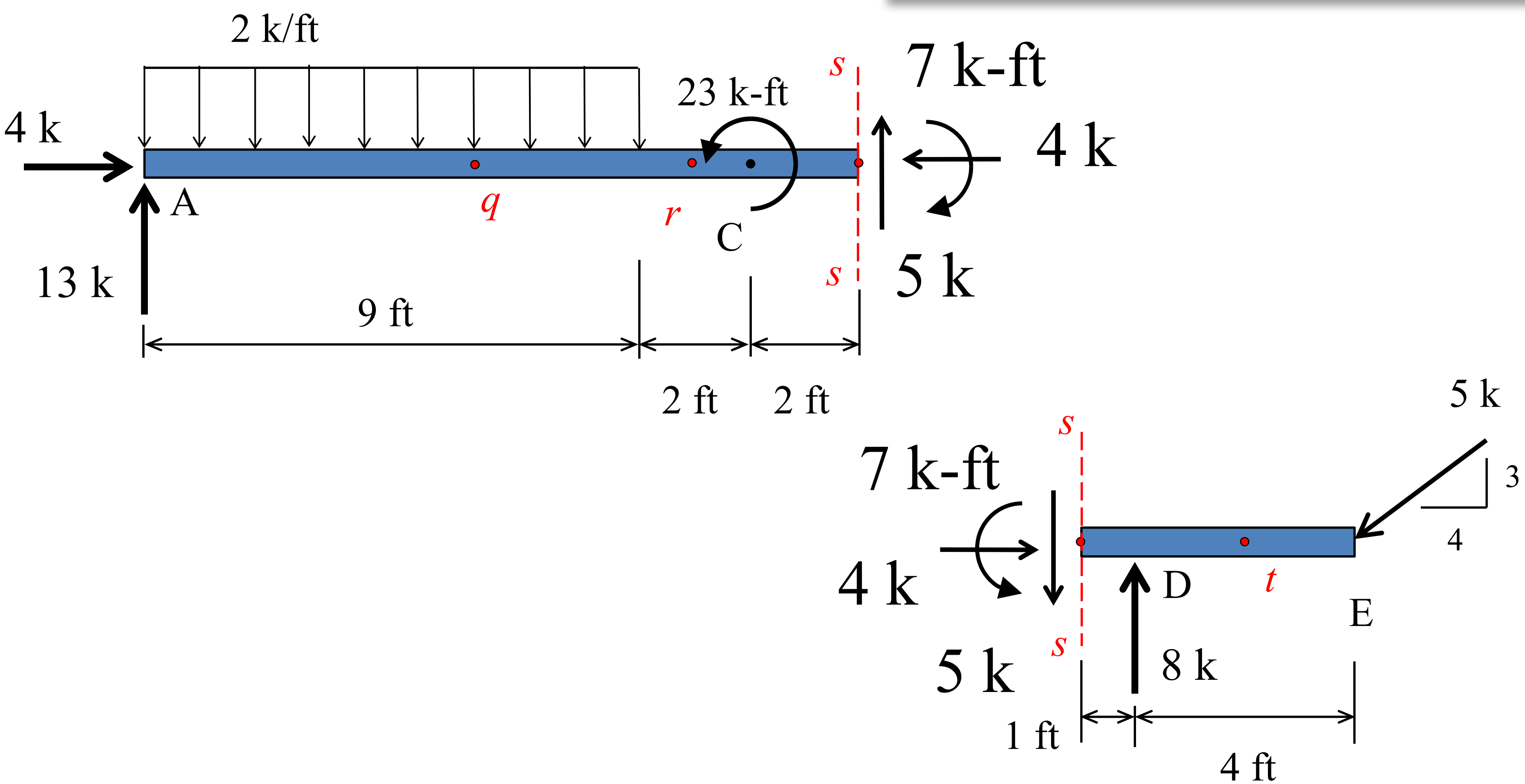
M_s = -7 k-ft

V_s = -5 k

F_s = -4 k

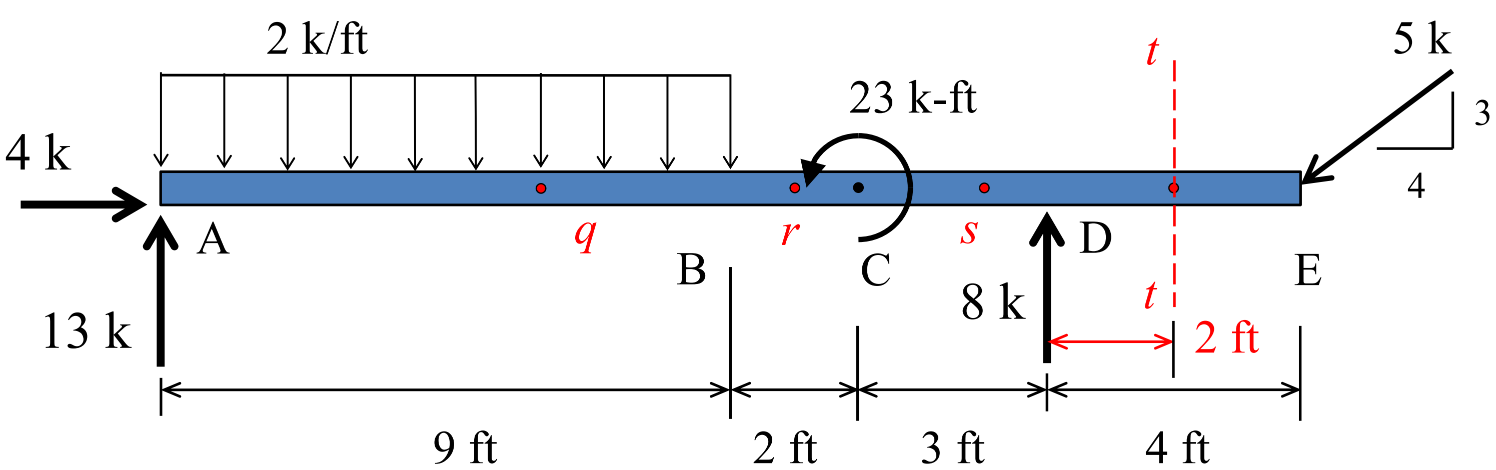
Internal Forces at Point *s*

Confirm that **both** segments are in equilibrium



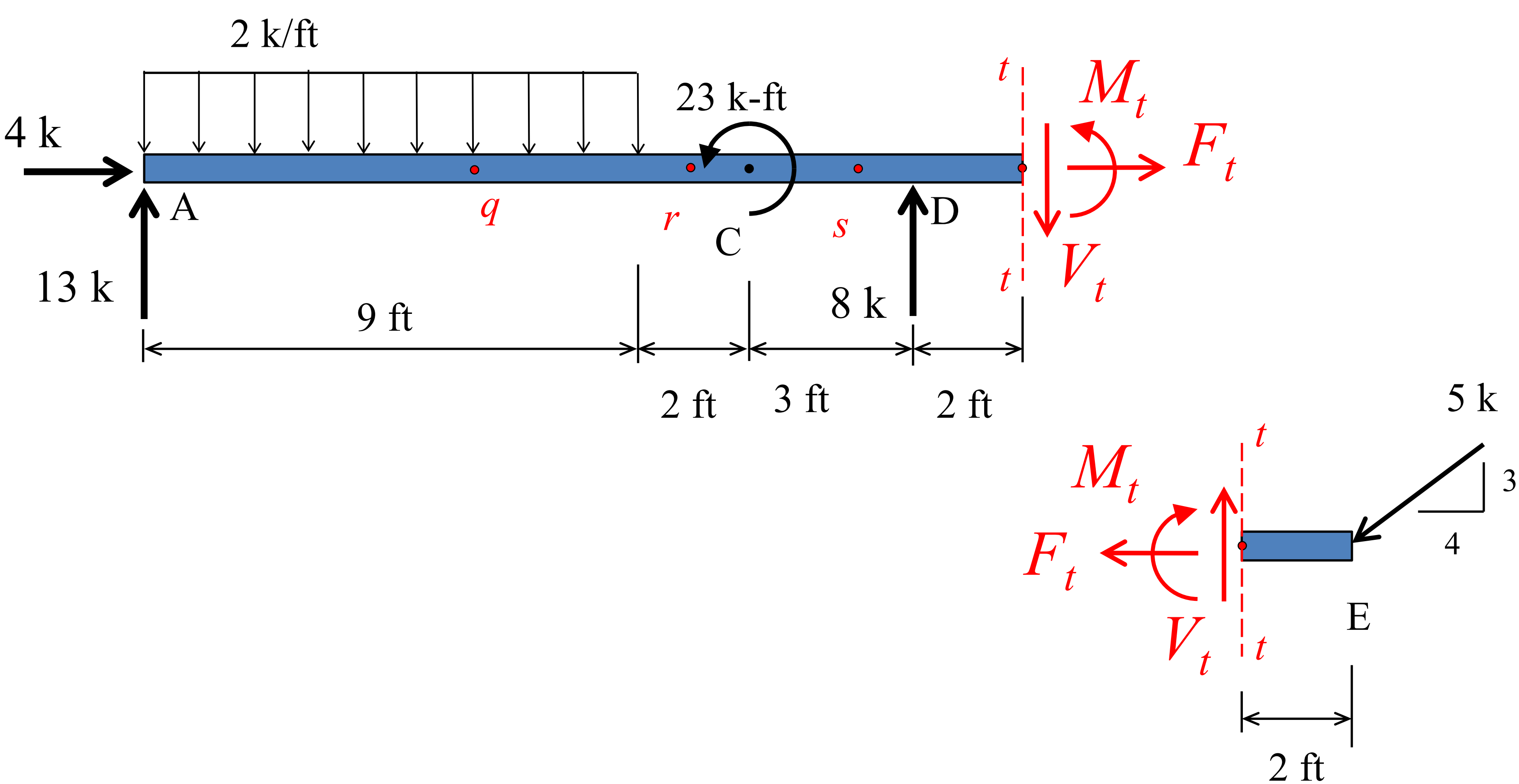
Find Internal Forces at Point *t*

Cut beam at point *t*



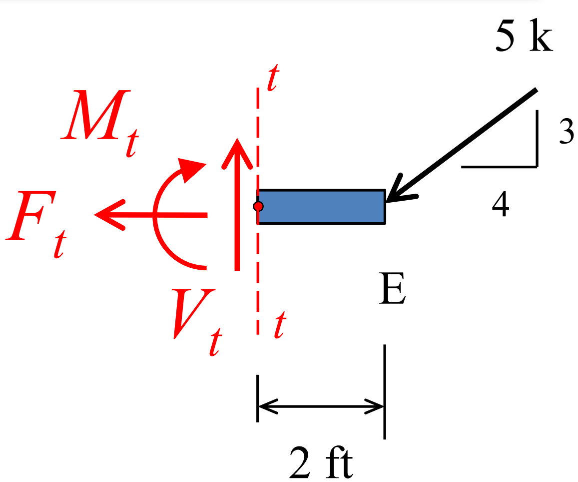
Find Internal Forces at Point *t*

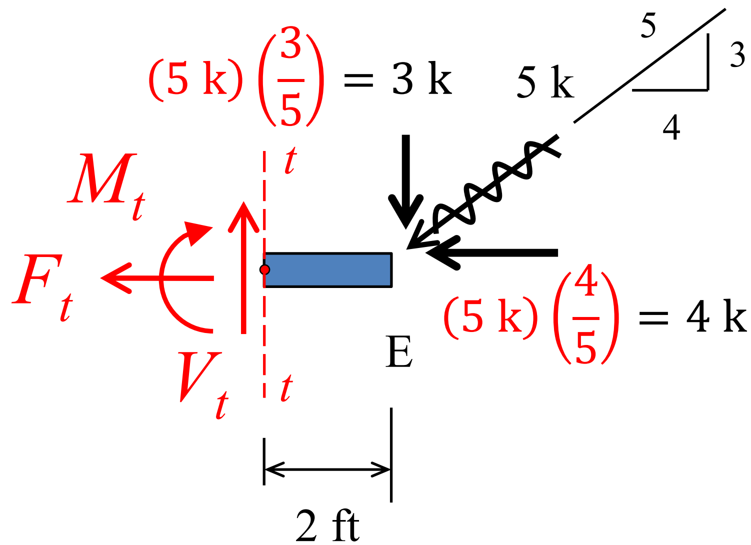
FBDs of Segments ABCD*t* and *t*E



Find Internal Forces at Point t

FBD of Segment tE





$$\circlearrowleft + \sum M_t = 0$$

$$+\uparrow \sum F_y = 0$$

$$\xrightarrow{+} \sum F_x = 0$$

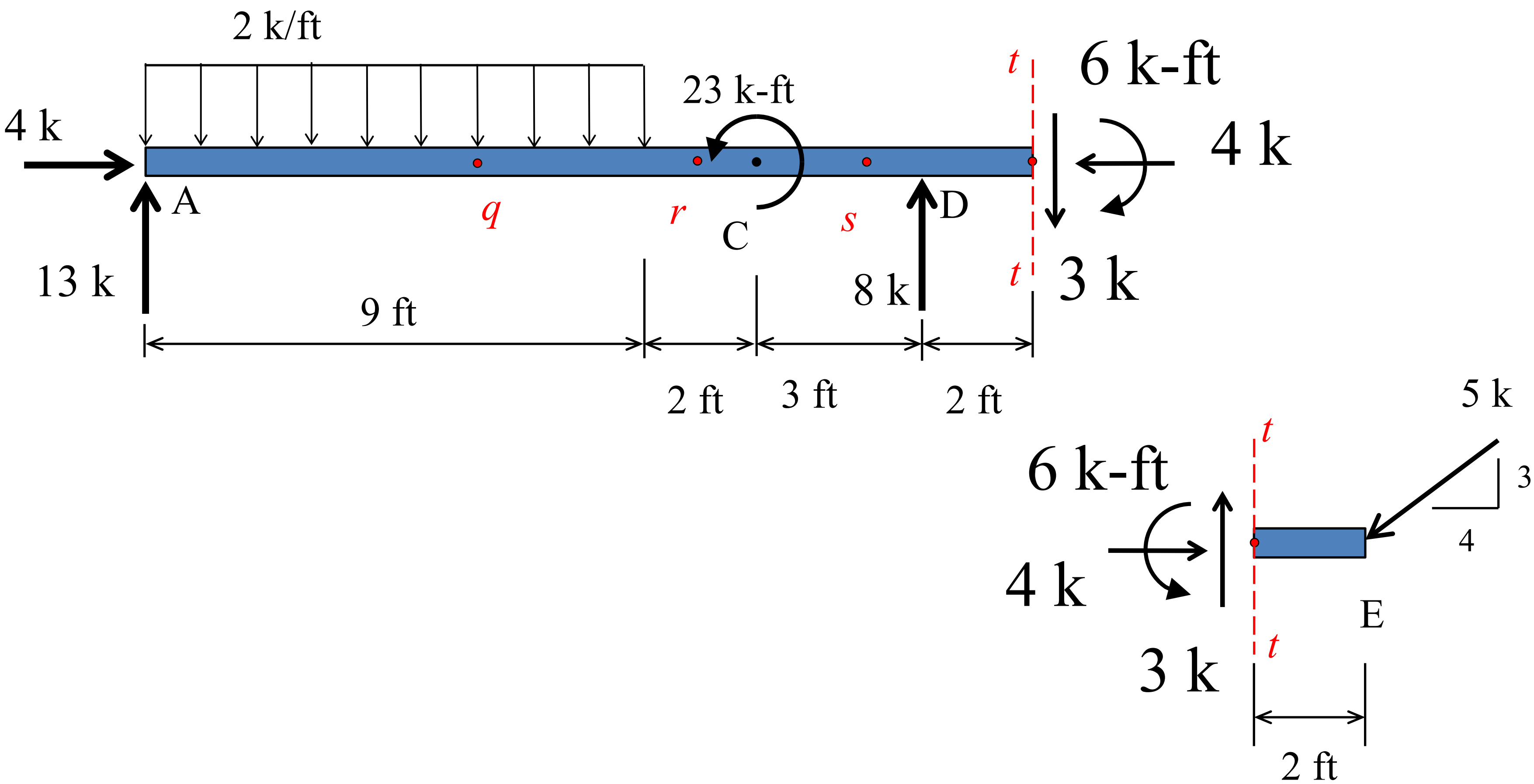
M_t = -6 k-ft

V_t = 3 k

F_t = -4 k

Internal Forces at Point *t*

Confirm that **both** segments are in equilibrium



Summary of Results