Equations of Equilibrium for Planar Structures

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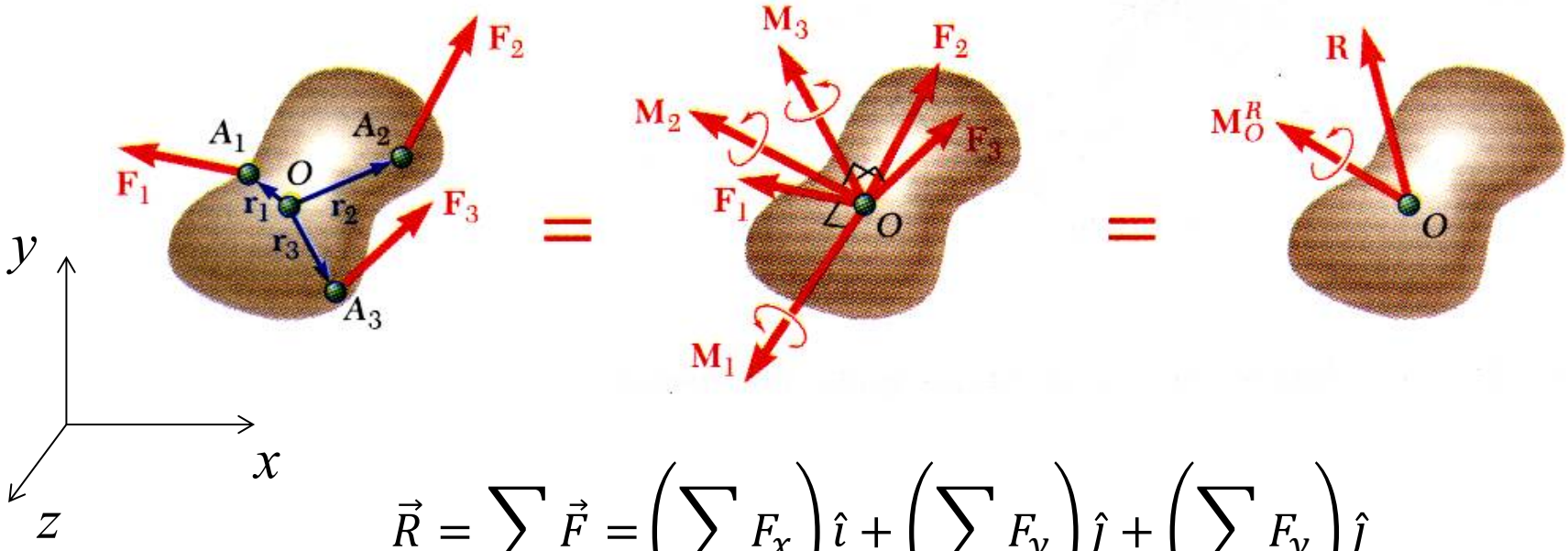
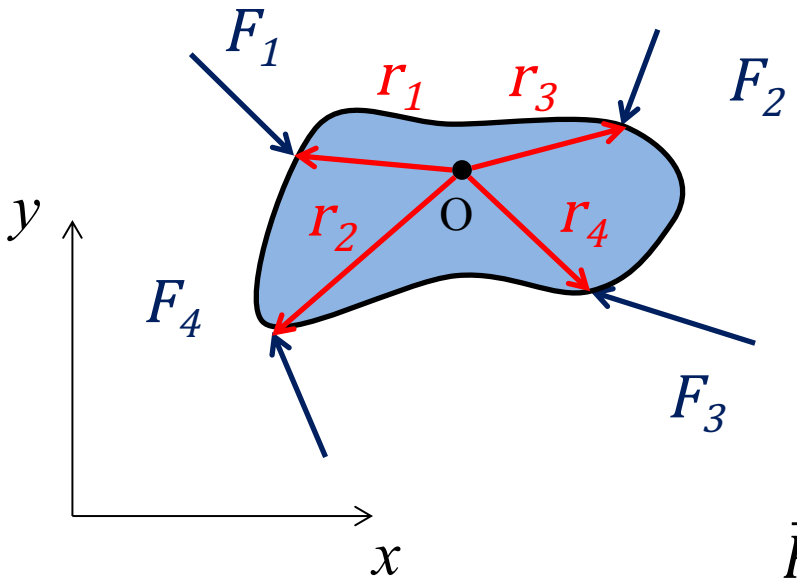
Recall that any force system acting on a body can be expressed as an equivalent force-couple system at any point



$$\vec{R} = \sum \vec{F} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath} + \left(\sum F_y\right)\hat{\jmath}$$

$$\vec{M}_O^R = \sum (\vec{r} \times \vec{F}) = \left(\sum M_x\right)\hat{\imath} + \left(\sum M_y\right)\hat{\jmath} + \left(\sum M_z\right)\hat{k}$$

For a two-dimensional (planar) body
in the *xy* plane the resultant moment vector will
always be in the *z* direction



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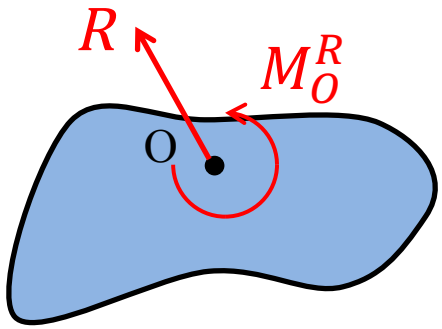


$$\vec{R} = \sum \vec{F} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath}$$

$$\vec{M}_O^R = \left(\sum (\vec{r} \times \vec{F})\right)\hat{k}$$

A body is in equilibrium if both the resultant force and resultant couple are equal to zero

Two vector equations of equlibrium

$$\vec{R} = \sum \vec{F} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath} + \left(\sum F_y\right)\hat{\jmath} = \vec{0}$$

$$\vec{M}_O^R = \sum (\vec{r} \times \vec{F}) = \left(\sum M_x\right)\hat{\imath} + \left(\sum M_y\right)\hat{\jmath} + \left(\sum M_z\right)\hat{k} = \vec{0}$$

Six scalar equations of equilibrium

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

For a general two-dimensional body, the six scalar equations simplify to three

Vector equations

$$\vec{R} = \sum \vec{F} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath}$$

$$\vec{M}_O^R = \left(\sum (\vec{r} \times \vec{F})\right)\hat{k}$$

Three scalar equations of equilibrium

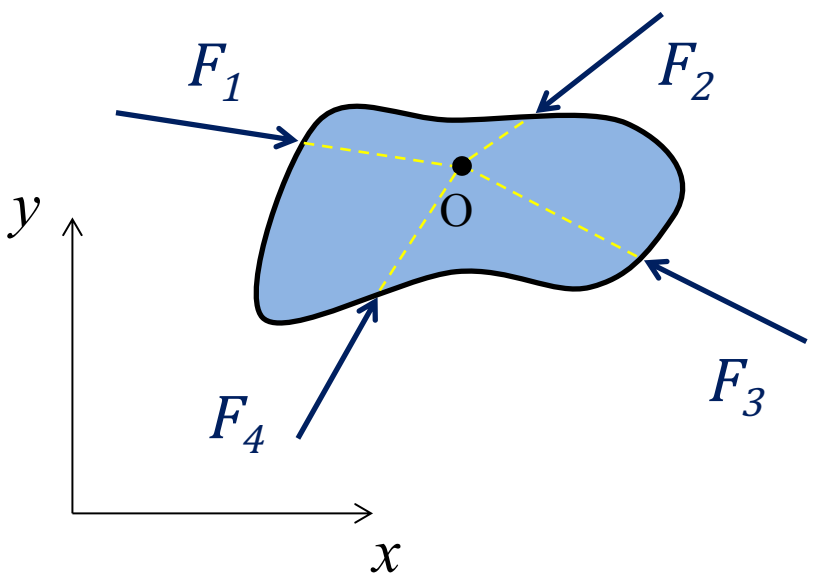
Can take moments about any point

$$\sum F_x = 0 \quad \sum F_y = 0$$

$$\sum M_z = 0$$

For a body where all of the forces are concurrent, rotational equilibrium is satisfied

Planar body with concurrent forces



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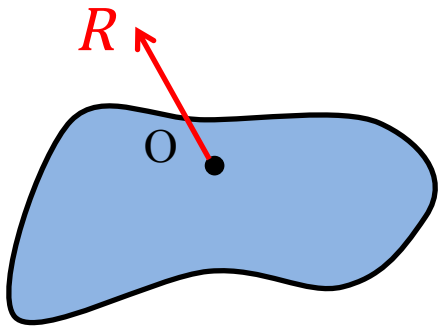

$$\vec{R} = \sum \vec{F} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath} = \vec{0}$$

Two scalar equations of equilibrium

$$\sum F_x = 0 \qquad \sum F_y = 0$$

General procedure for the Analysis of Bodies in Static Equilibrium

- Choose the free body to isolate;
- Draw a **Free Body Diagram (FBD)** of the body;
  - Isolate the body from all of its surroundings,
  - Magnitudes and directions of all known and unknown forces acting on the body should be included and clearly indicated,
  - Indicate dimensions on the FBD,
- Write the **equations of equilibrium** and solve the equations for the unknown quantities.