Shear Force and Bending Moment Diagrams for a Beam

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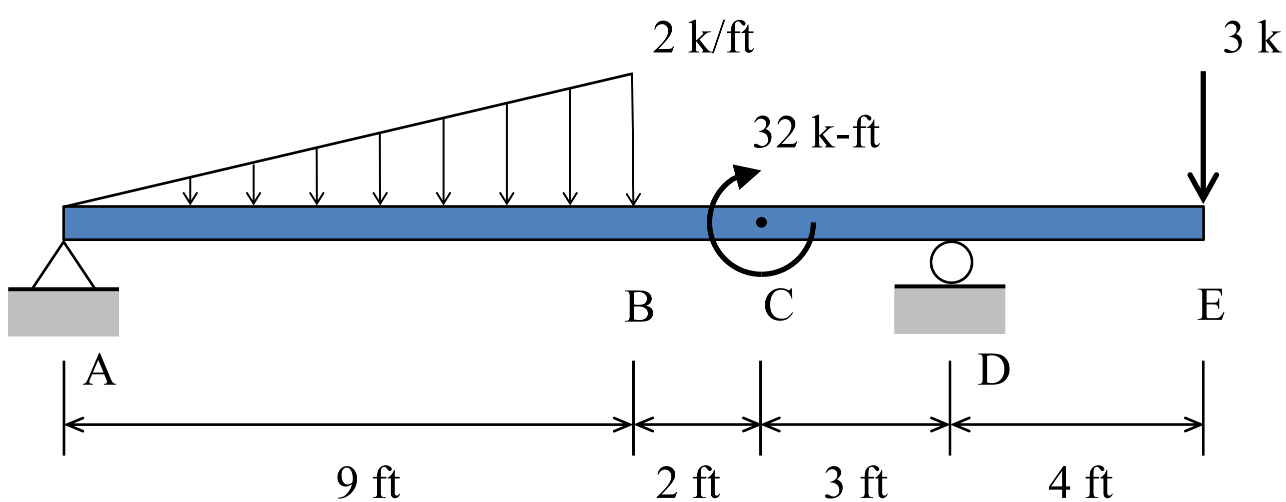
General procedure for the construction of internal force diagrams

1. Find all of the external forces and draw the external force diagram;
2. Choose a sign convention for each diagram;
3. If necessary, choose a reference coordinate system:
4. Use equilibrium analysis or differential and integral relationships to construct internal force functions;
   - Cut structure at appropriate sections,
   - The FBD on either side of the cut may be analyzed,
   - Indicate unknown internal forces consistent with the chosen sign convention,
   - Plot the internal force function for each segment,
5. Check each diagram for errors;
   - Check discontinuities at location of applied forces in shear diagram,
   - Check discontinuities at location of applied moment in moment diagram,
   - Check differential and integral relationships between distributed load, shear, and bending moment.

Shear and Bending Moment Diagram Example

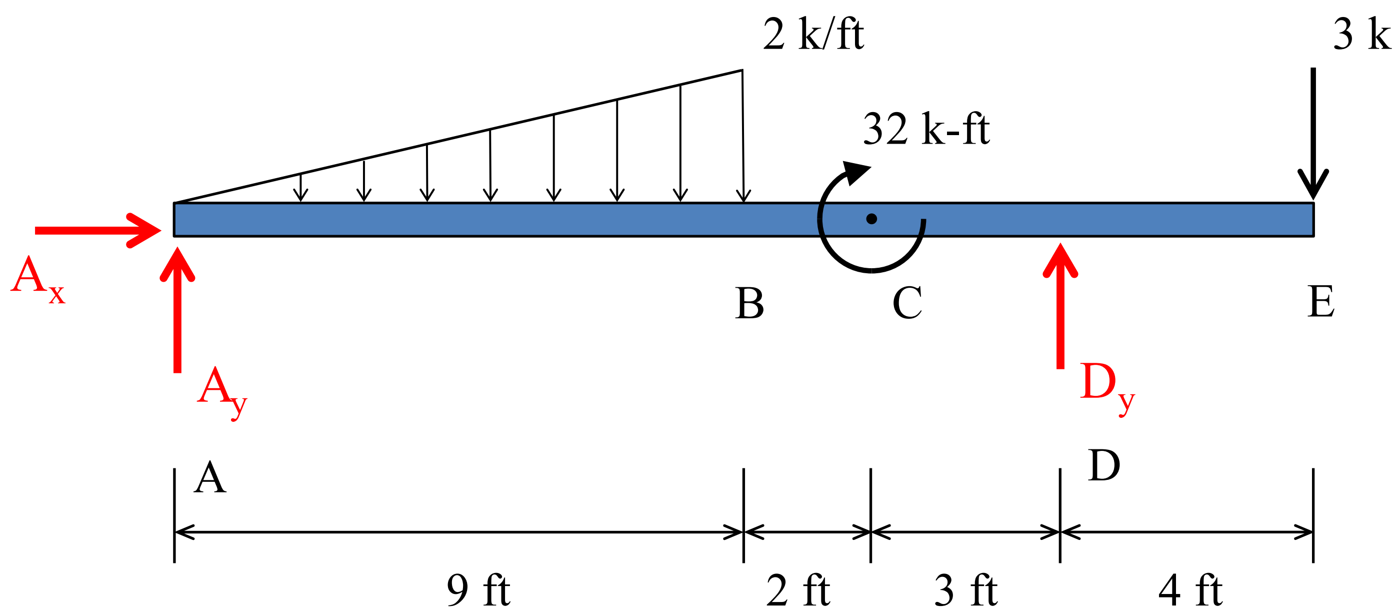
A beam is supported by a pin support at point A and extends over a roller support at point D. The beam is and subjected to a linearly varying load from A to B, a point moment at point D a point load at point E as shown.

Draw the diagram and the bending moment diagram for the beam. Label all local maximum and minimum values and their locations and show your sign convention for each diagram.

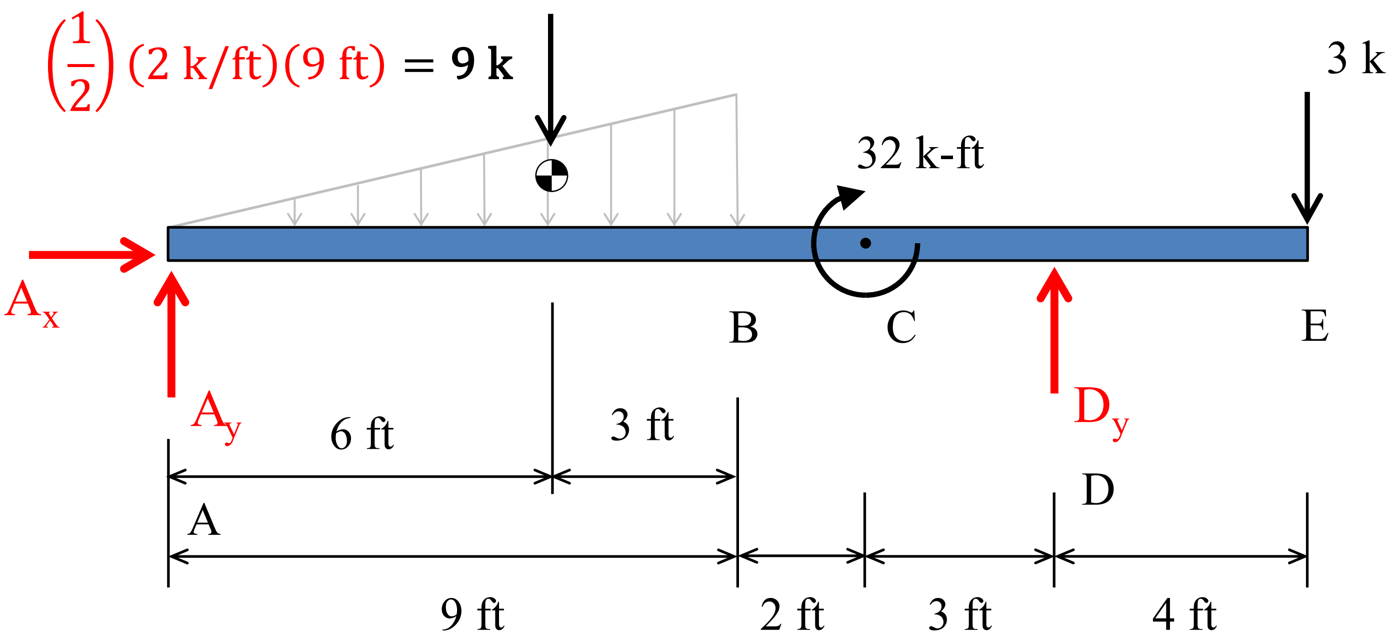


1. Find all of the external forces and draw the external force diagram;

FBD of beam



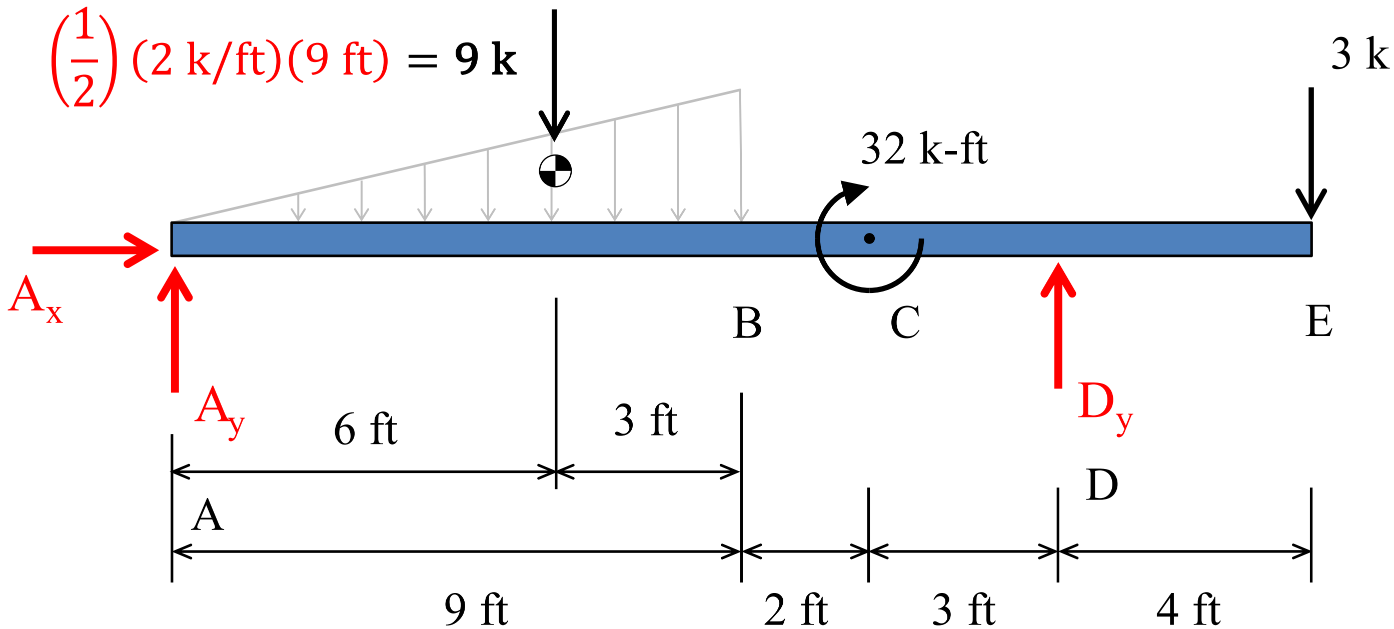
Find support reactions using equations of equilibrium



$$\circlearrowleft + \sum M_A = 0$$

$D_y = 10$ k

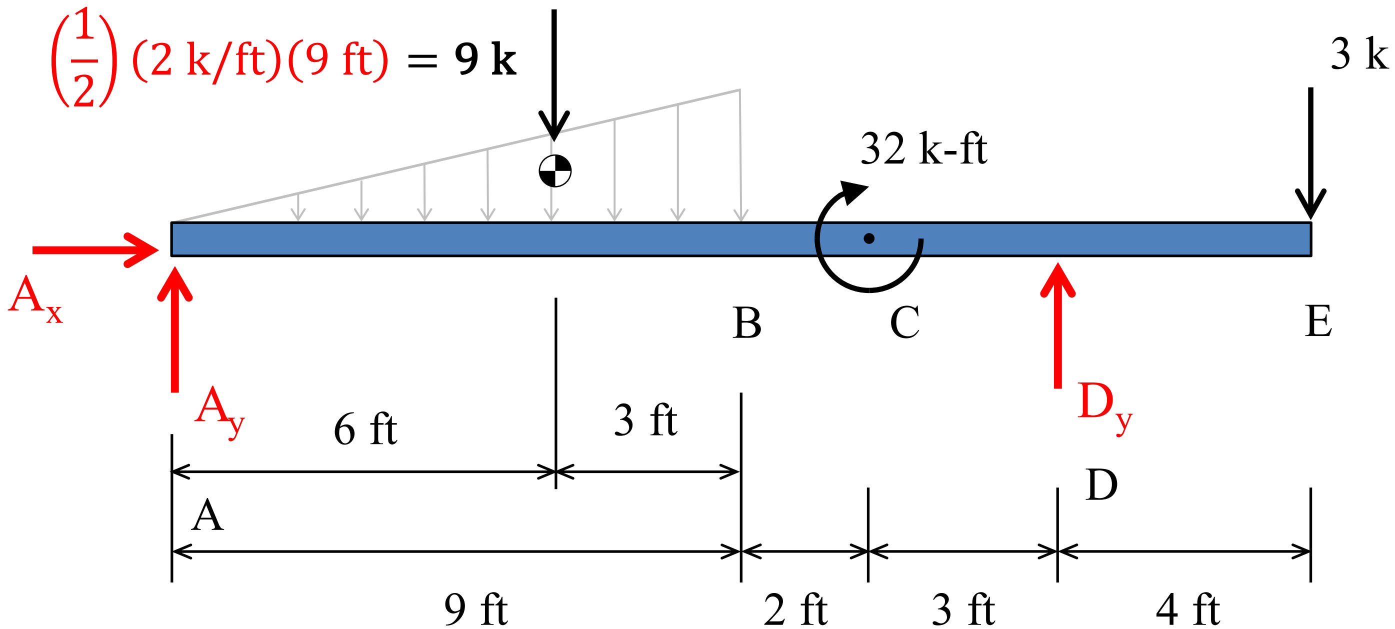
Find support reactions using equations of equilibrium



$$+\uparrow \sum F_y = 0$$

$A_y = 2$ k

Find support reactions using equations of equilibrium

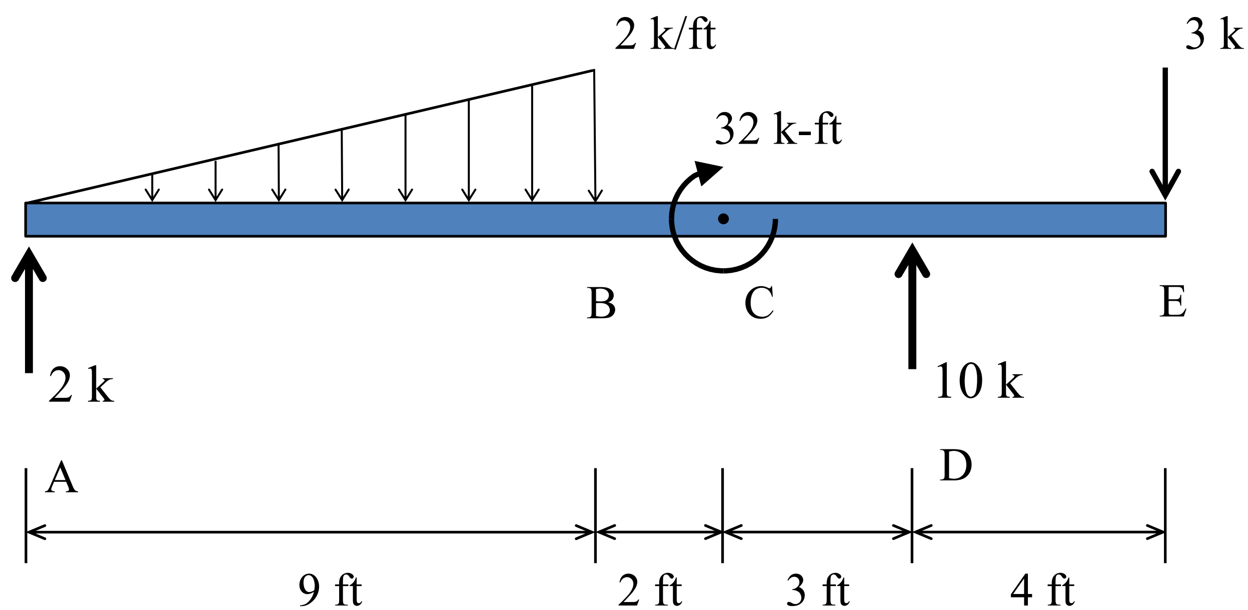


$$\xrightarrow{+} \sum F_x = 0$$

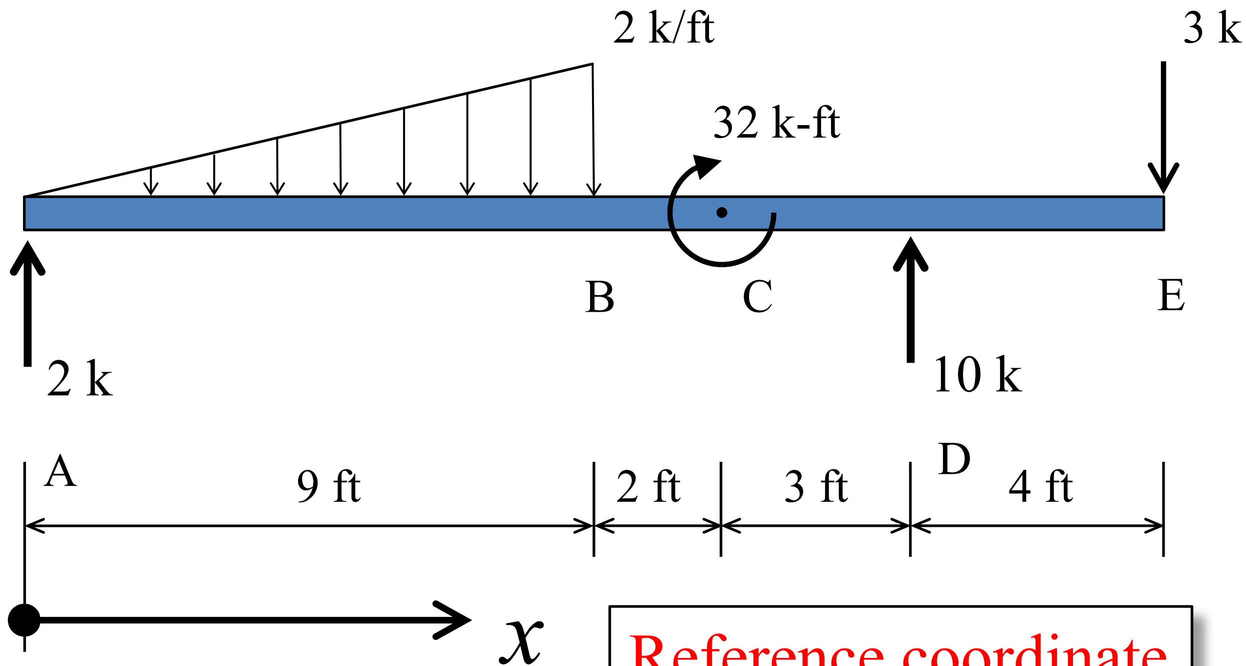
$A_x = 0$

External force diagram

FBD of beam showing all known external forces

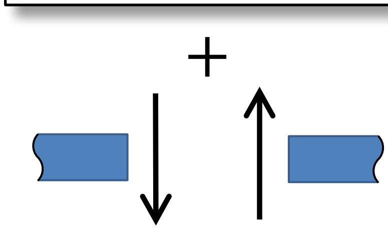


2. Choose a sign convention for each diagram;
3. If necessary, choose a reference coordinate system

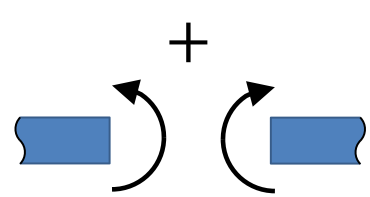


Reference coordinate

Positive shear

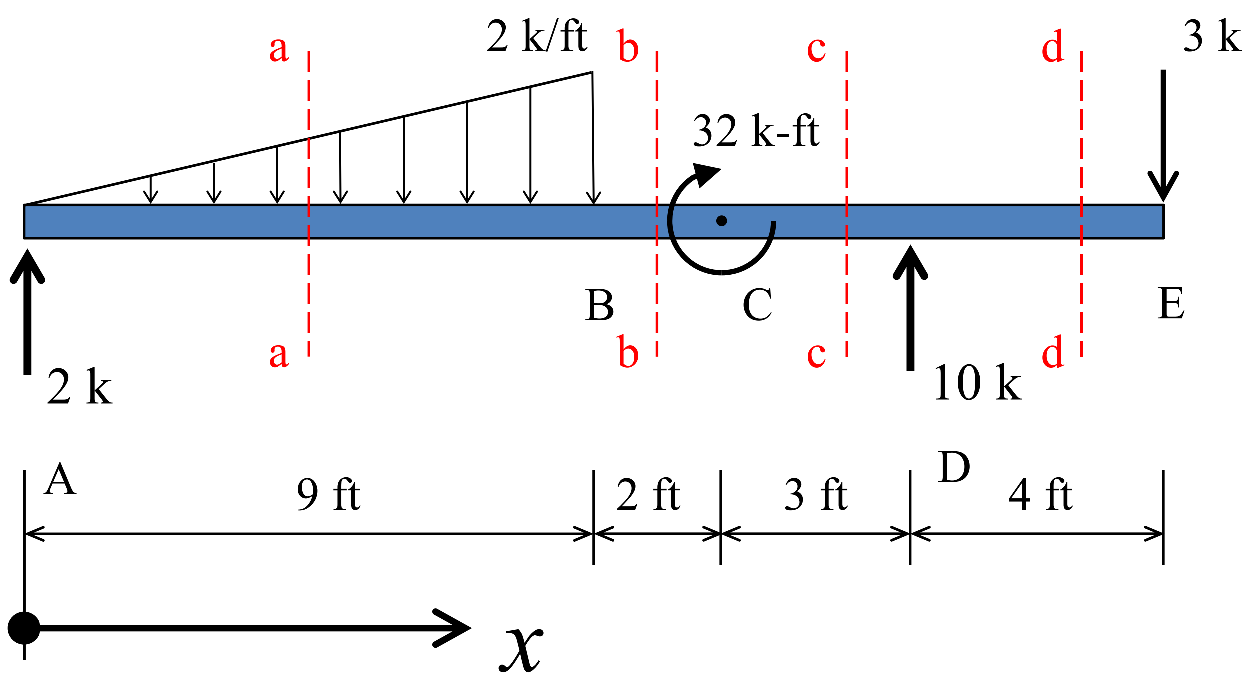


Positive bending moment

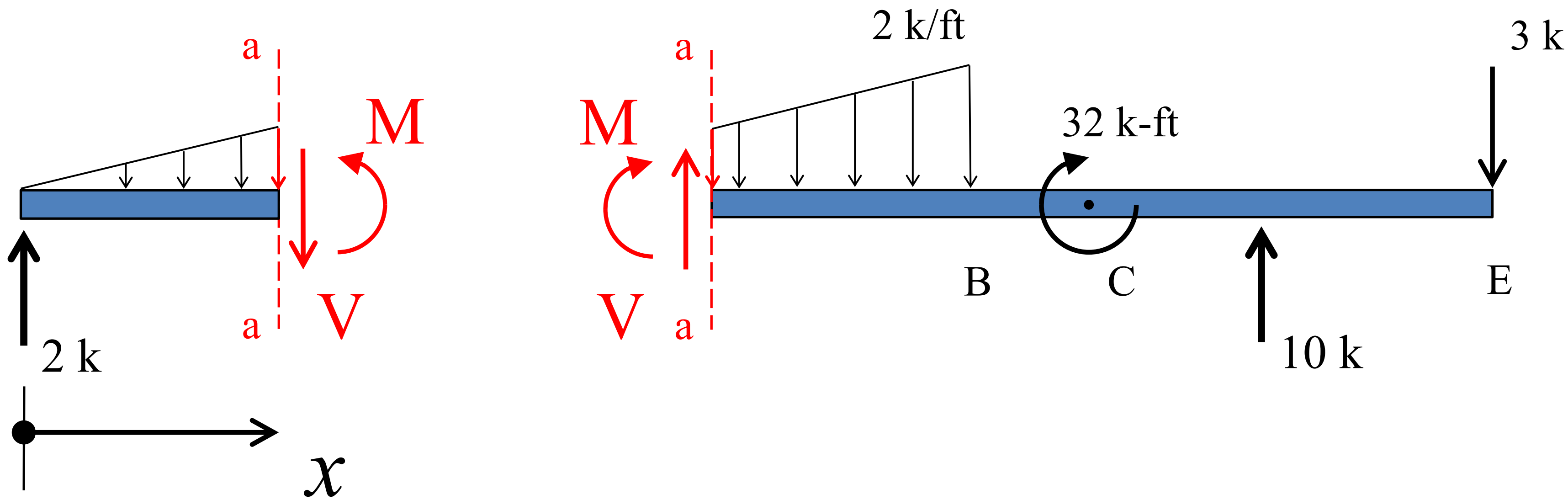


4. Use equilibrium analysis or differential and integral relationships to construct internal force functions;

- Cut structure at appropriate sections,
- The FBD on either side of the cut may be analyzed,
- Indicate unknown internal forces consistent with the chosen sign convention,
- Plot the internal force function for each segment,



Analysis of Segment AB
FBD of sections on either side of cut at section a-a

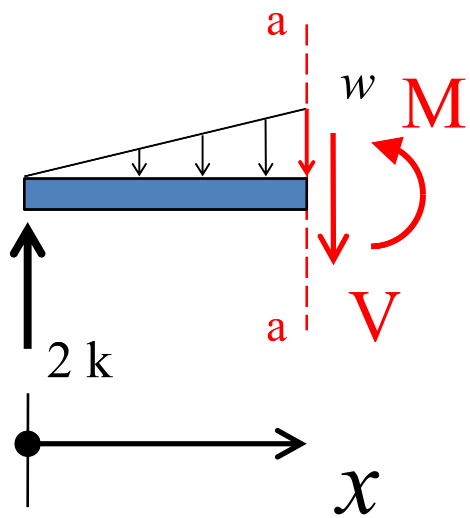


Notes

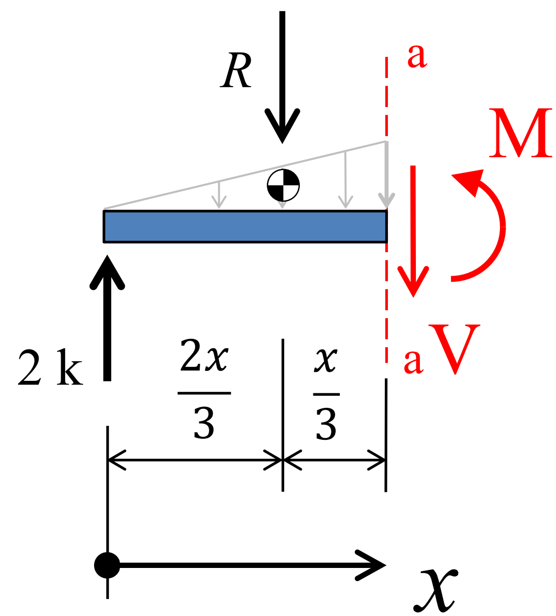
- The FBD on either side of the cut may be analyzed;
- Unknown internal forces are consistent with the chosen sign convention;
- For this segment the FBD to the left of section a-a is probably the best choice to analyze.

FBD of section to the left of cut a-a

$$w = \left(\frac{2\ \text{k/ft}}{9\ \text{ft}}\right)(x) = \left(\frac{2}{9}\right)x$$

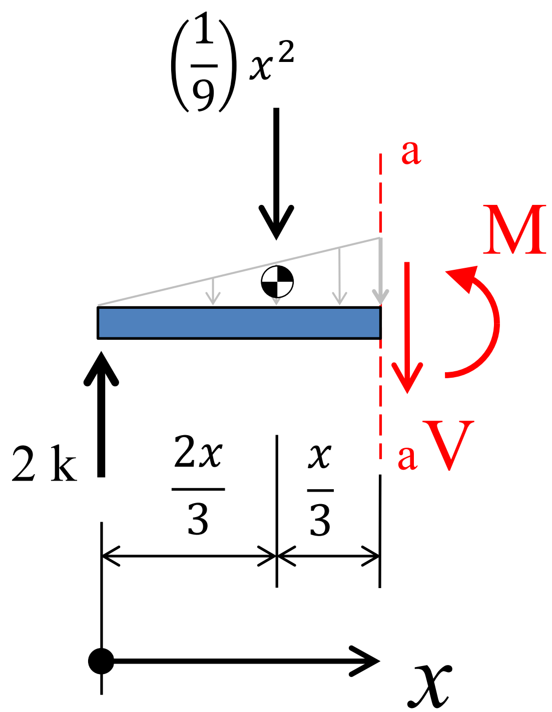


$$R = \left(\frac{1}{2}\right)\left[(x)\left(\frac{2}{9}\right)\right](x) = \left(\frac{1}{9}\right)x^2$$



Notes
- Find the intensity of the distributed load as a function of x;
- Replace the distributed load as an equivalent point load;
- Two unknowns (M, V) and two equations of equilibrium available.

Solve for V and M functions for segment AB



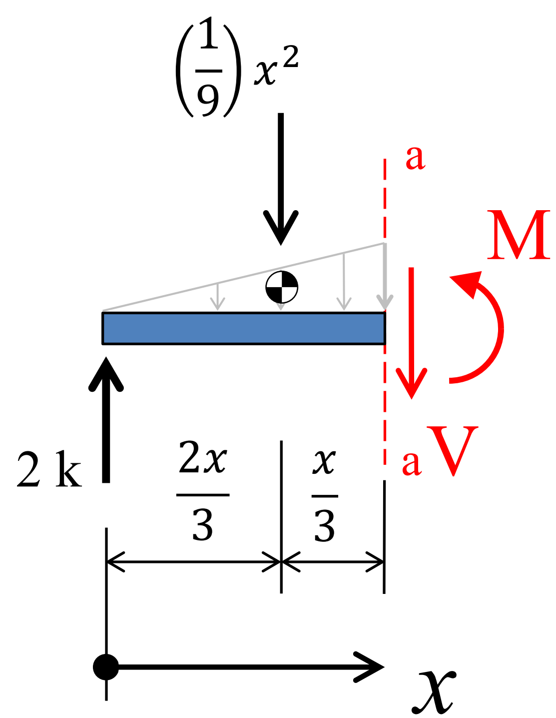
$$+\uparrow \sum F_y = 0$$

$$0 \leq x \leq 9$$

$$V(0) = V_A = 2 \text{ k}$$
$$V(9) = V_B = -7 \text{ k}$$

$$V = 2 - \left(\frac{1}{9}\right)x^2$$

Solve for V and M functions for segment AB



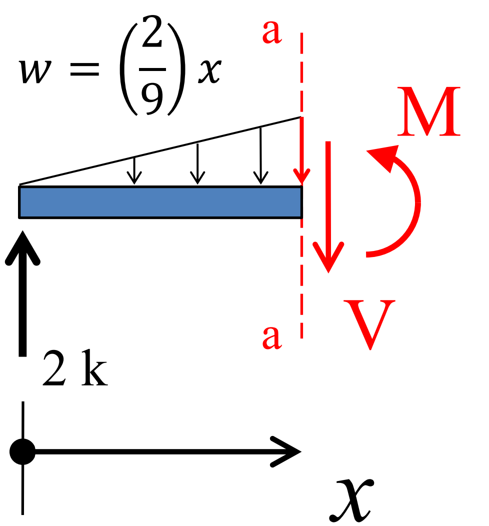
$$\circlearrowleft + \sum M_a = 0$$

$$0 \leq x \leq 9$$

$$M(0) = M_A = 0$$
$$M(9) = M_B = -9 \text{ k-ft}$$

$$M = 2x - \left(\frac{1}{27}\right)x^3$$

Relationships between *w*, *V*, and *M*



$0 \leq x \leq 9$ ft

$$w = -\left(\frac{2}{9}\right)x$$

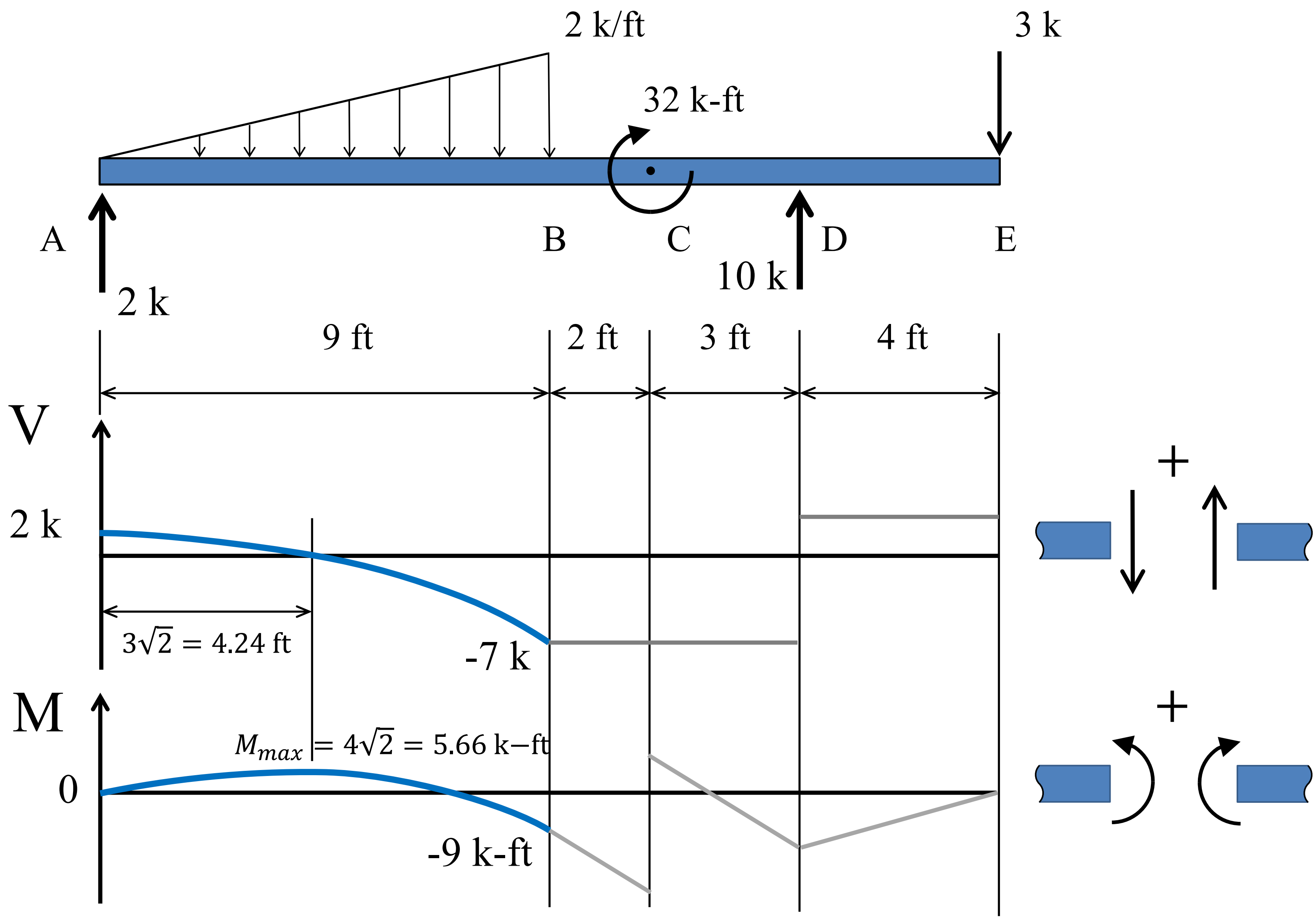
$$V = 2 - \left(\frac{1}{9}\right)x^2$$

$$M = 2x - \left(\frac{1}{27}\right)x^3$$

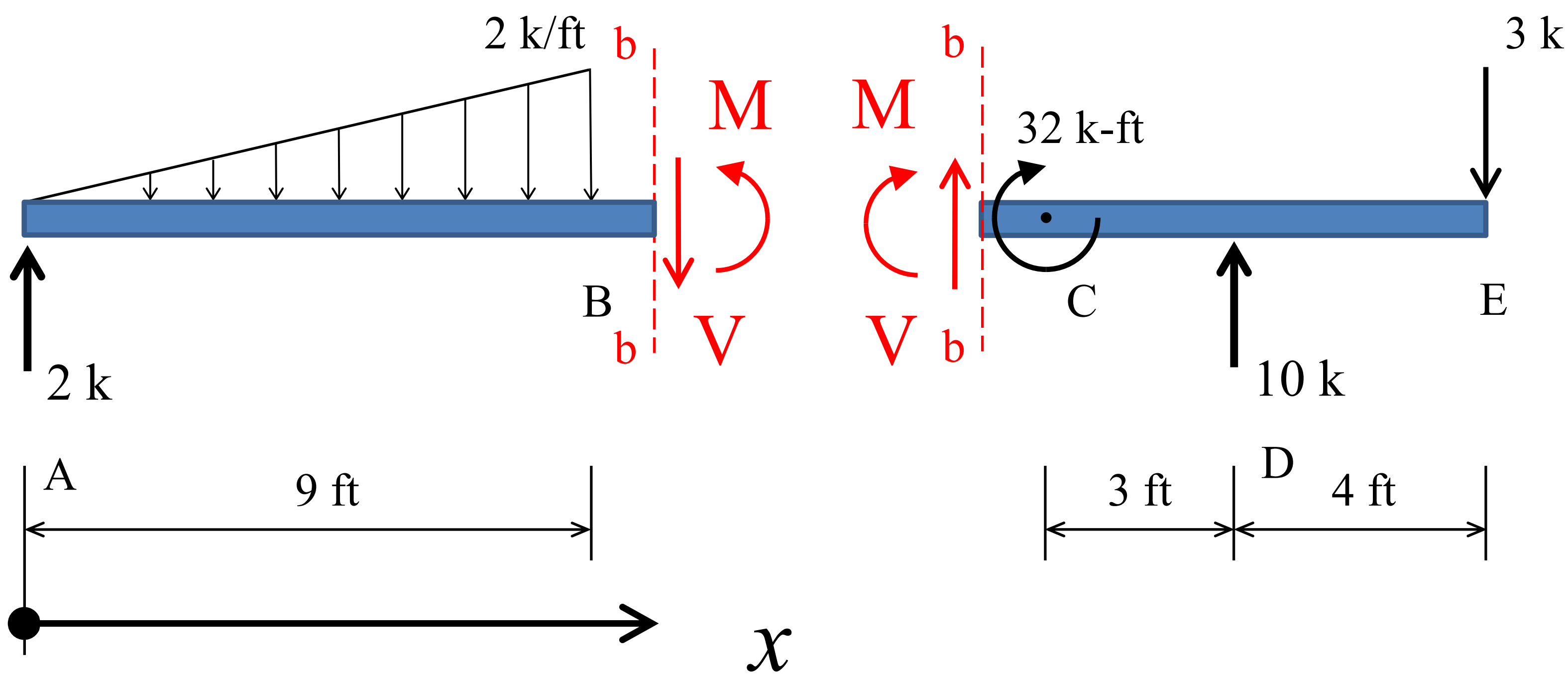
$$\boxed{\frac{dV}{dx} = -\left(\frac{1}{9}\right)2x = w}$$

$$\boxed{\frac{dM}{dx} = 2 - \left(\frac{1}{27}\right)3x^2 = V}$$

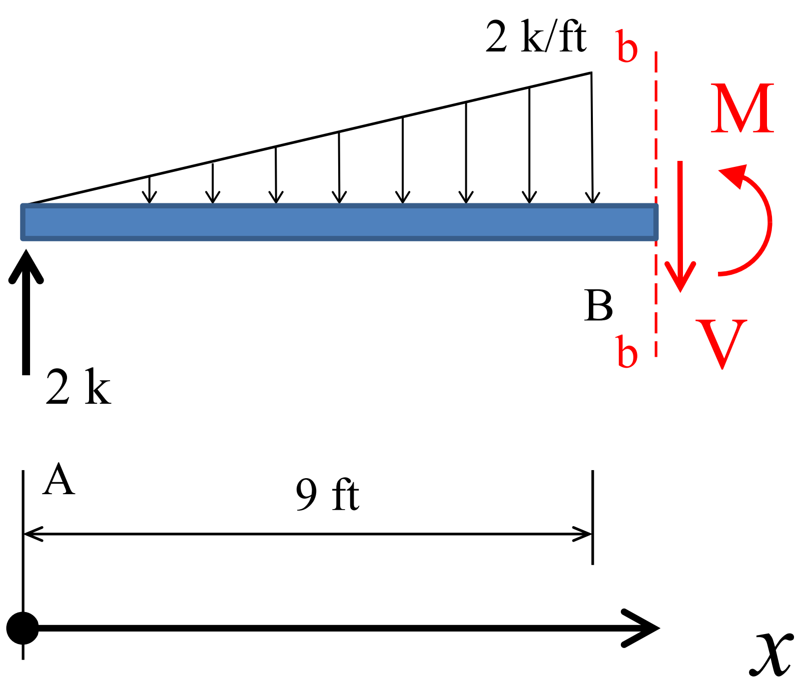
Plot V and M functions for segment AB

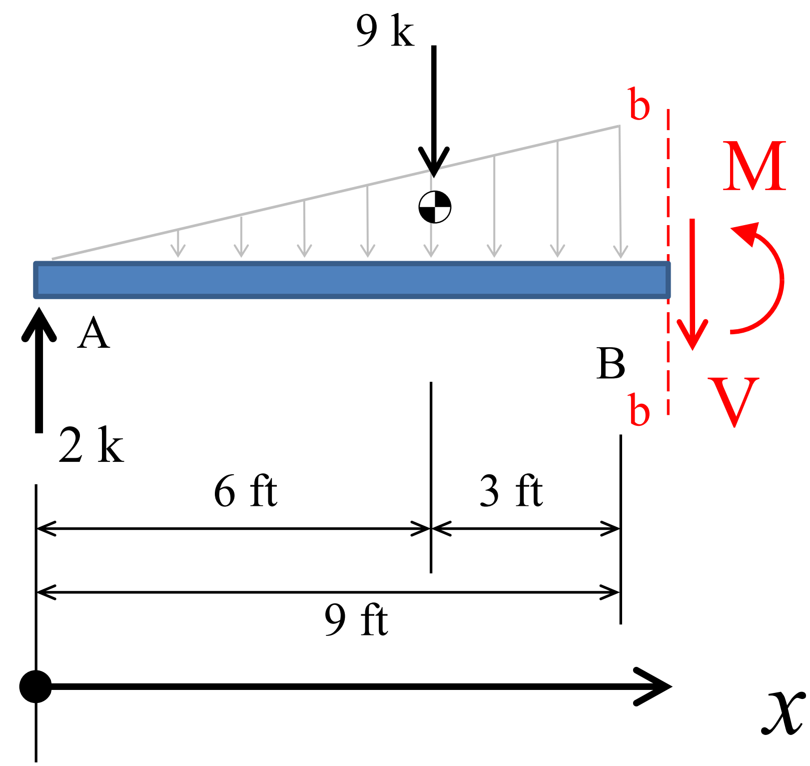


Analysis of Segment BC
FBD of sections on either side of cut at section b-b

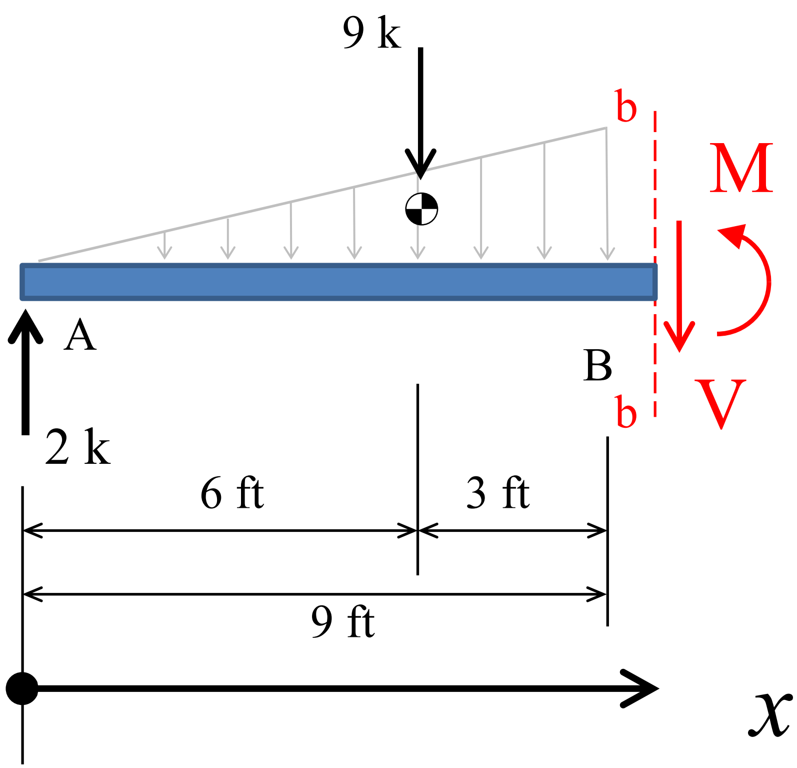


FBD of section to the left of cut b-b





Solve for V and M functions for segment BC



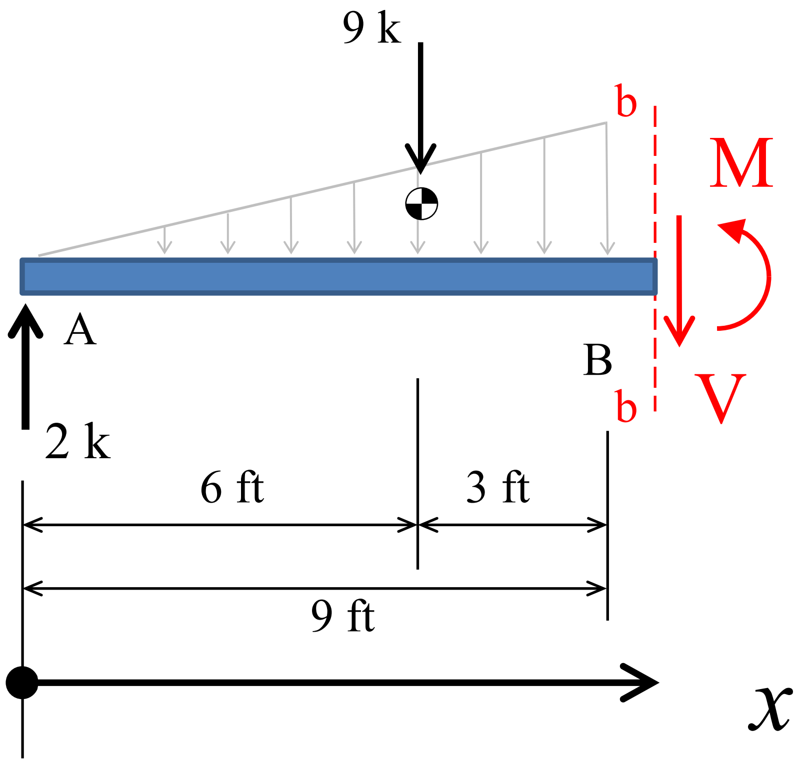
$$+\uparrow \sum F_y = 0$$

$$9 \leq x \leq 11$$

$$V(9) = V_{B+} = -7 \text{ k}$$
$$\text{V}(11) = V_{C-} = -7 \text{ k}$$

$$V = -7$$

Solve for V and M functions for segment BC



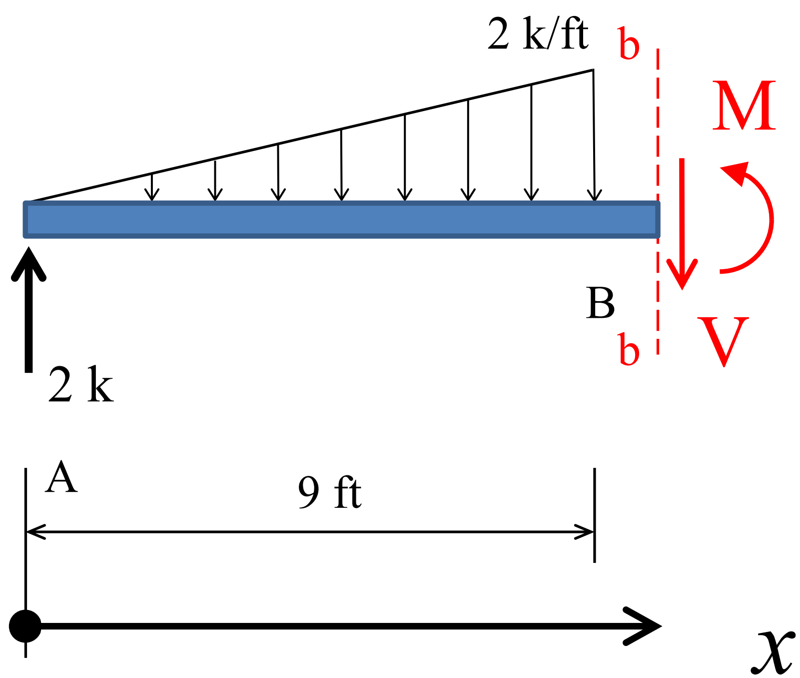
$$\circlearrowleft + \sum M_b = 0$$

$$9 \leq x \leq 11$$

$$M(9) = M_B = -9 \text{ k–ft}$$
$$M(11) = M_{C-} = -23 \text{ k-ft}$$

$$M = 54 - 7x$$

Relationships between w, V, and M



$$w = 0$$

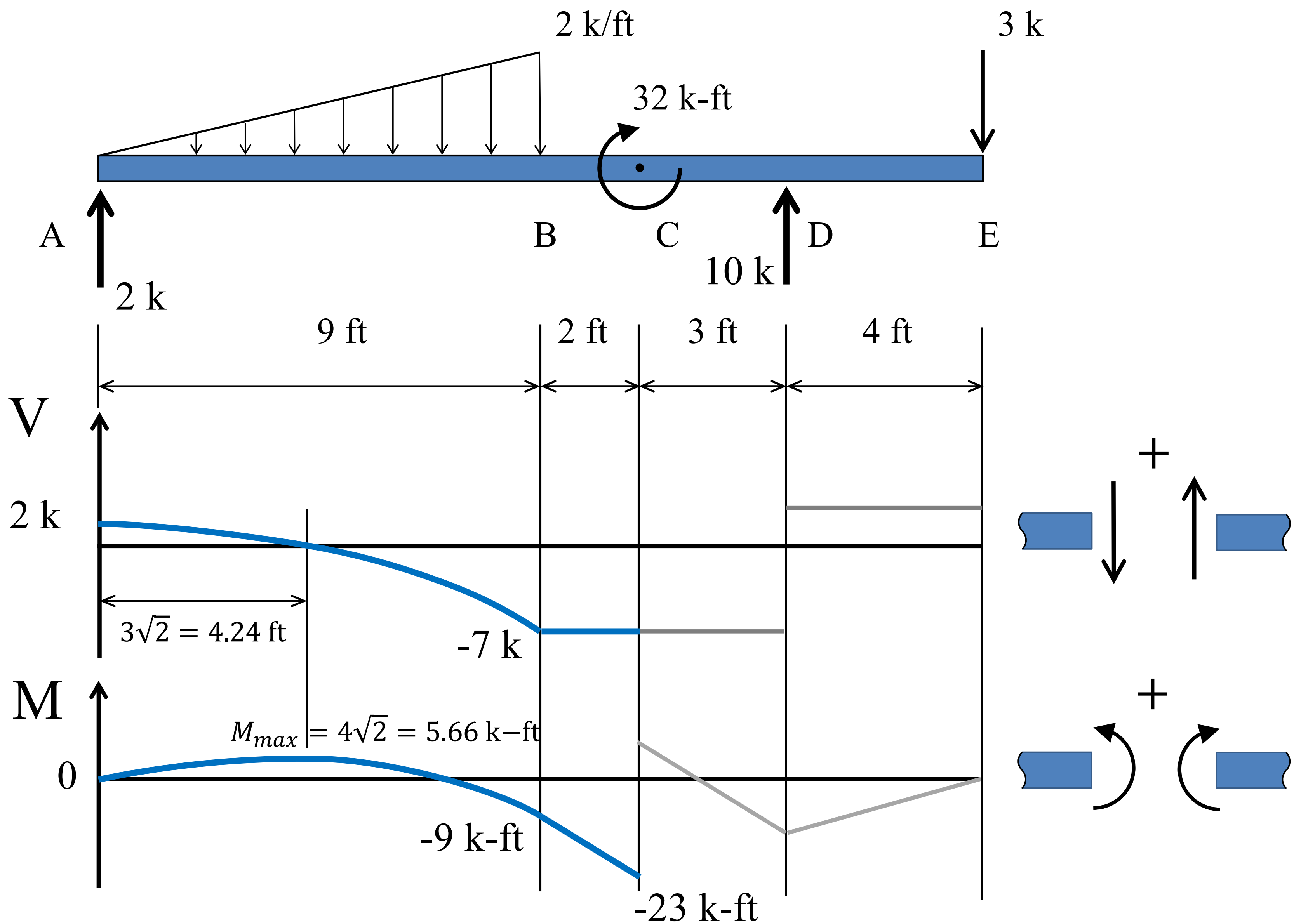
$$V = -7$$

$$M = 54 - 7x$$

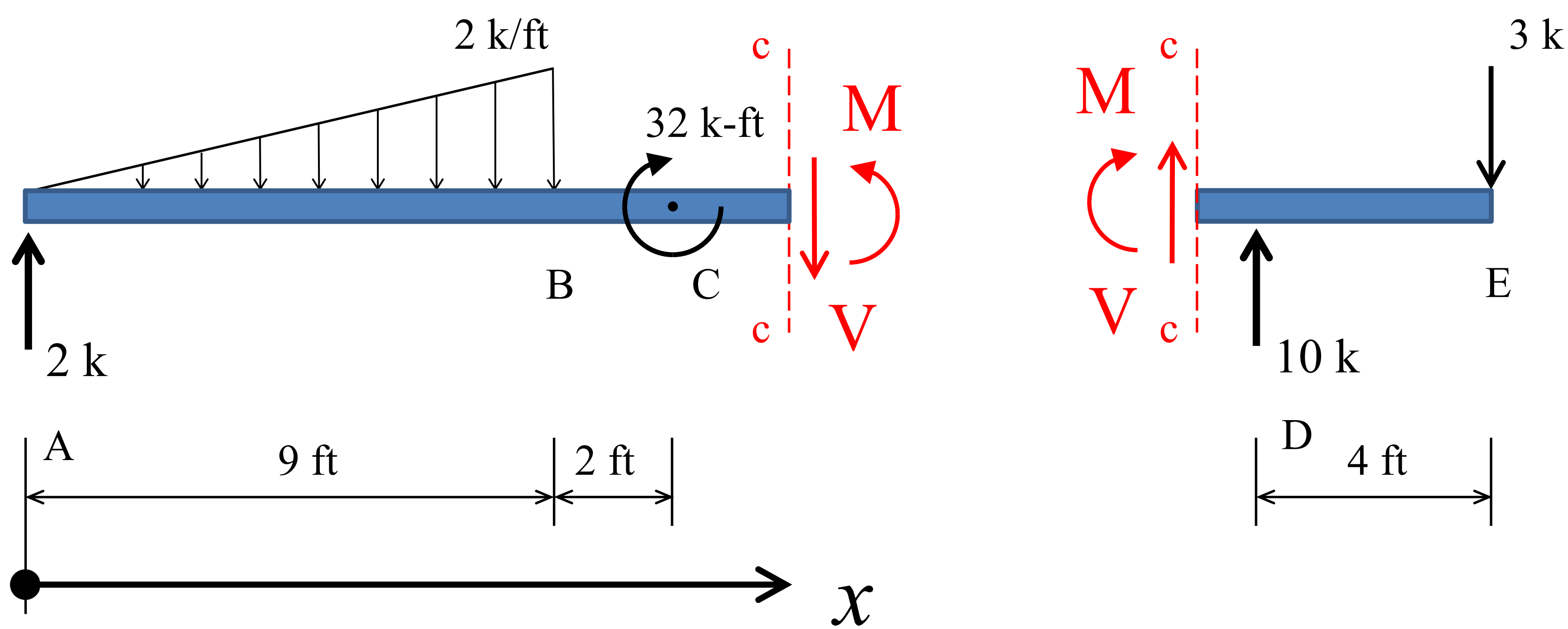
$$\boxed{\frac{dV}{dx} = 0 = w}$$

$$\boxed{\frac{dM}{dx} = -7 = V}$$

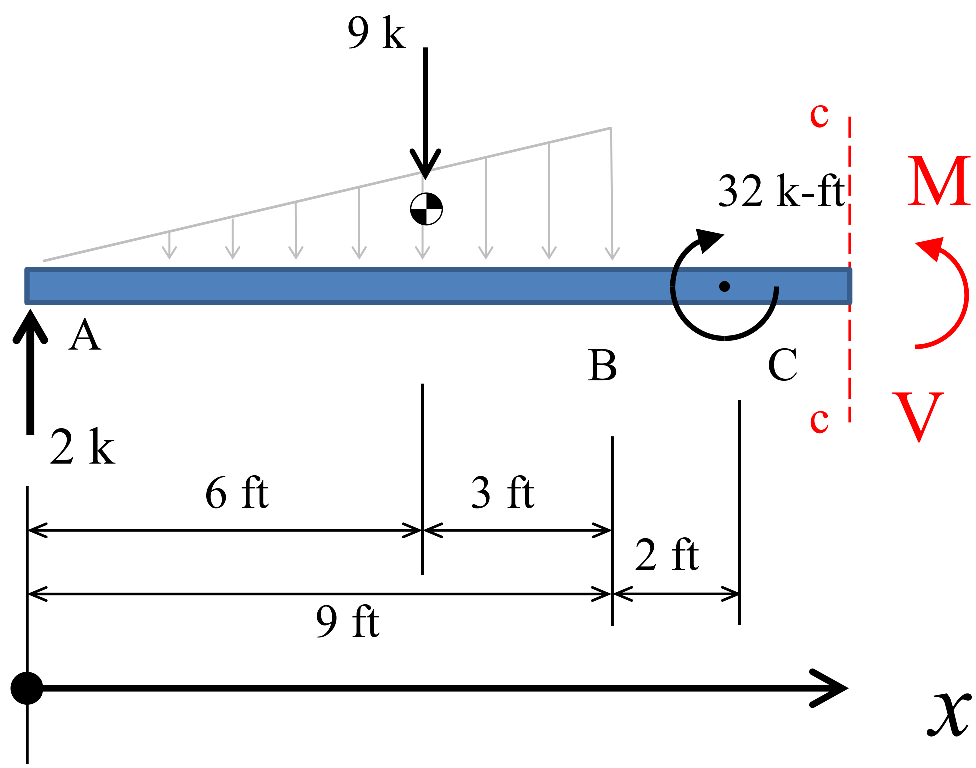
Plot V and M functions for segment BC



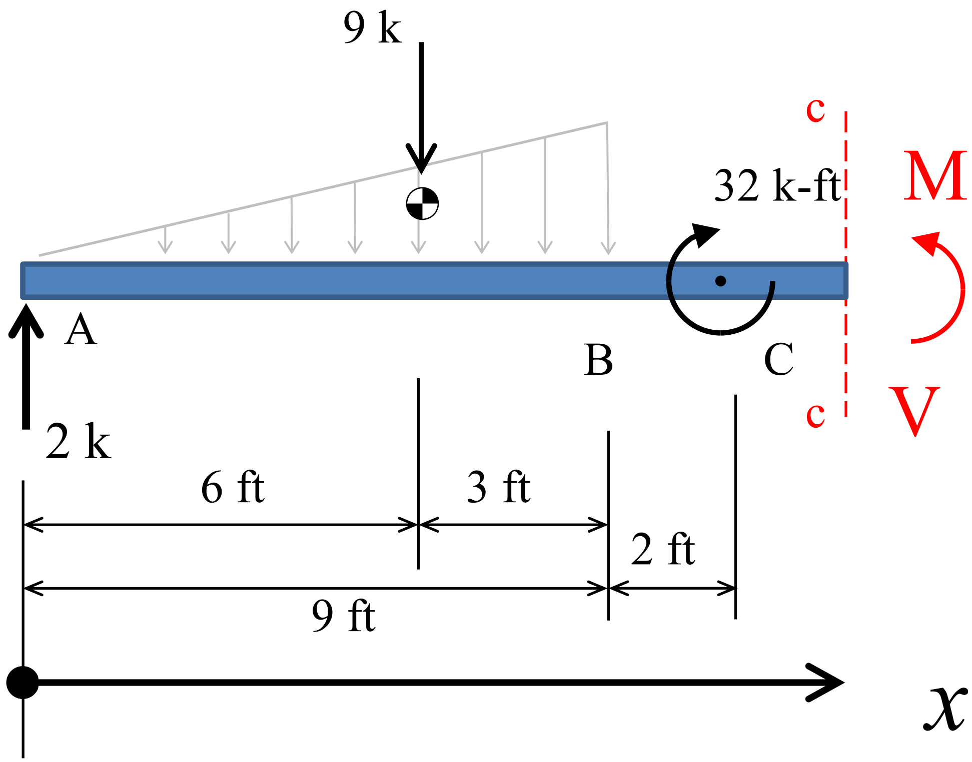
Analysis of Segment CD
FBD of sections on either side of cut at section c-c



FBD of section to the left of cut c-c



Solve for V and M functions for segment CD



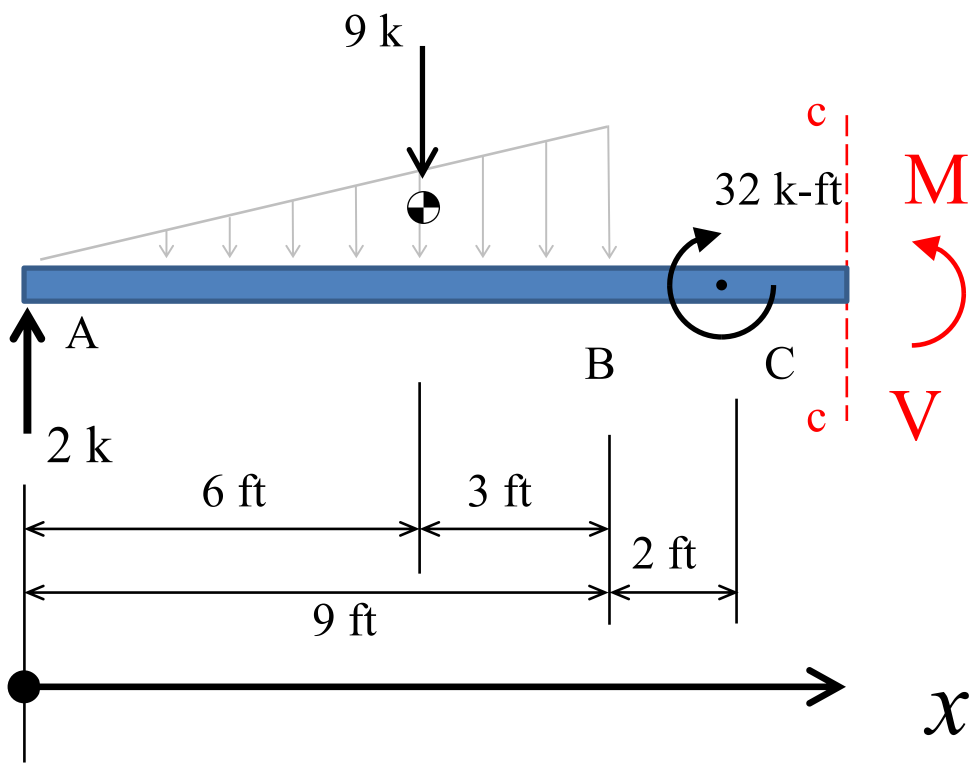
$$+\uparrow \sum F_y = 0$$

$$11 \leq x \leq 14$$

$$V(11) = V_{C+} = -7 \text{ k}$$
$$\text{V}(14) = V_{D-} = -7 \text{ k}$$

$$V = -7$$

Solve for V and M functions for segment CD



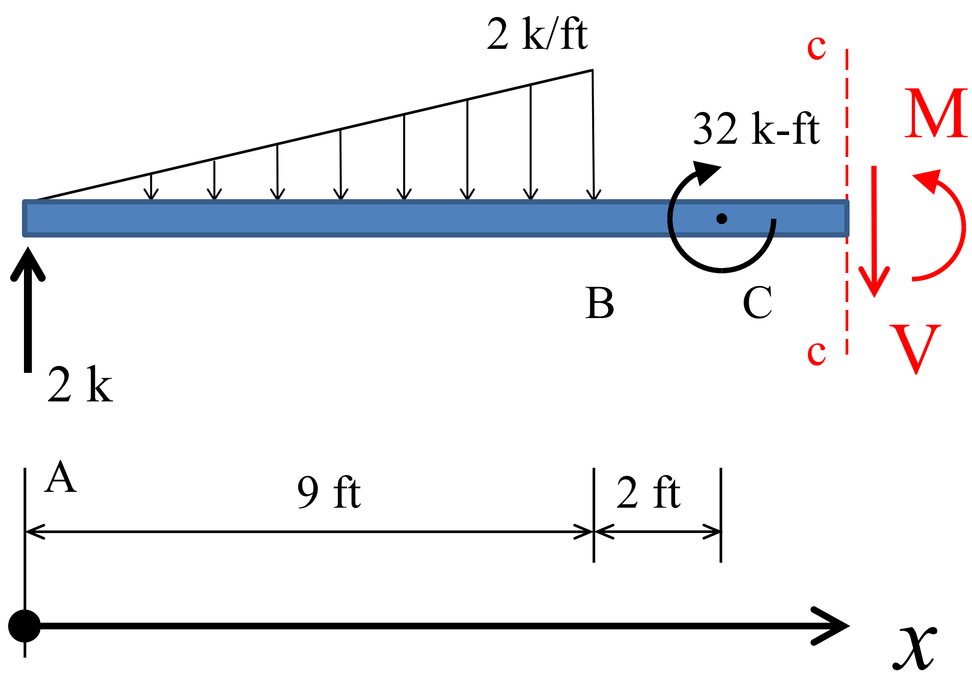
$$\circlearrowleft + \sum M_c = 0$$

$$11 \leq x \leq 14$$

$$M(11) = M_{C+} = 9 \text{ k-ft}$$
$$M(14) = M_D = -12 \text{ k-ft}$$

$$M = 86 - 7x$$

Relationships between *w*, *V*, and *M*



$$w = 0$$

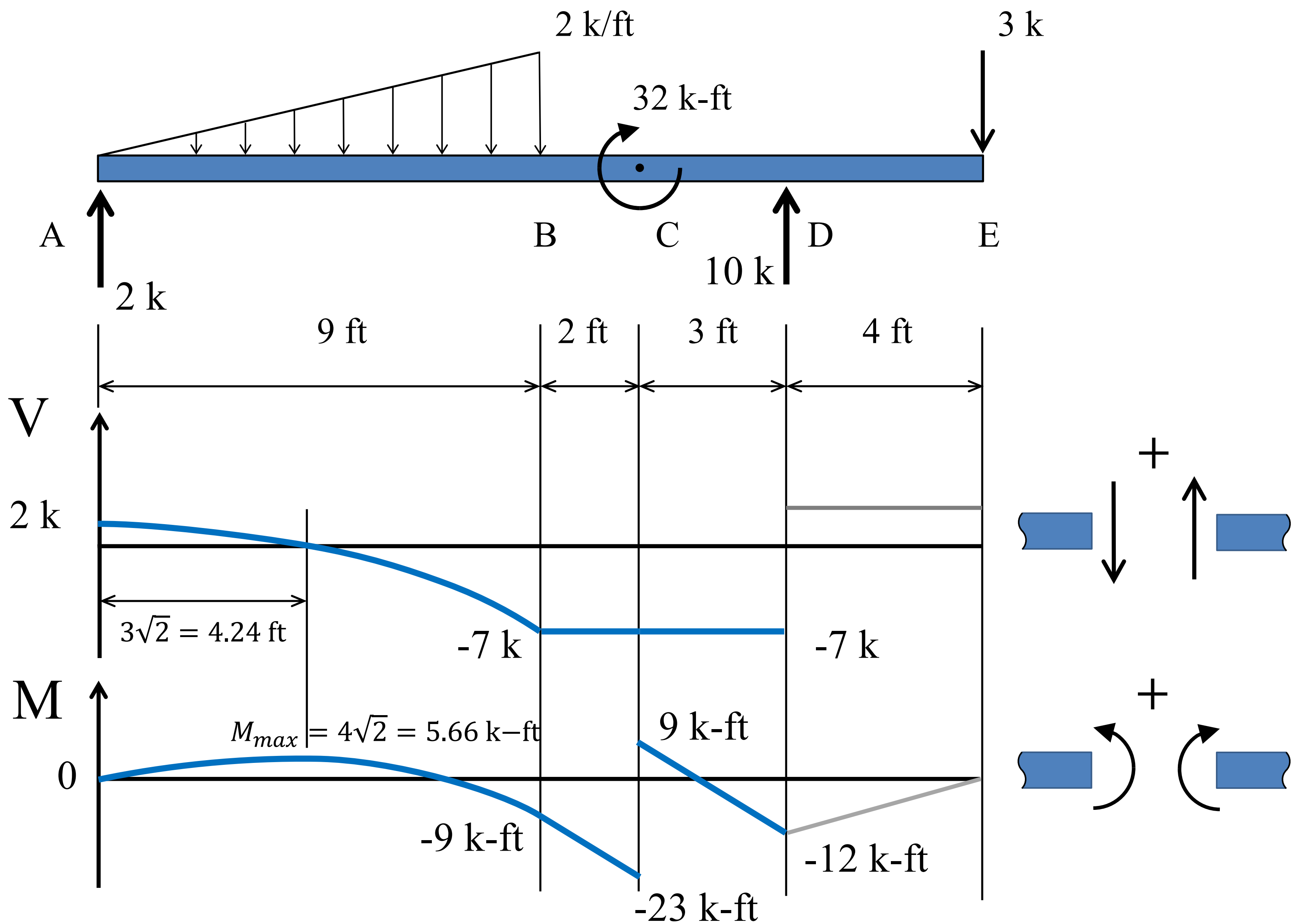
$$V = -7$$

$$M = 86 - 7x$$

$$\boxed{\frac{dV}{dx} = 0 = w}$$

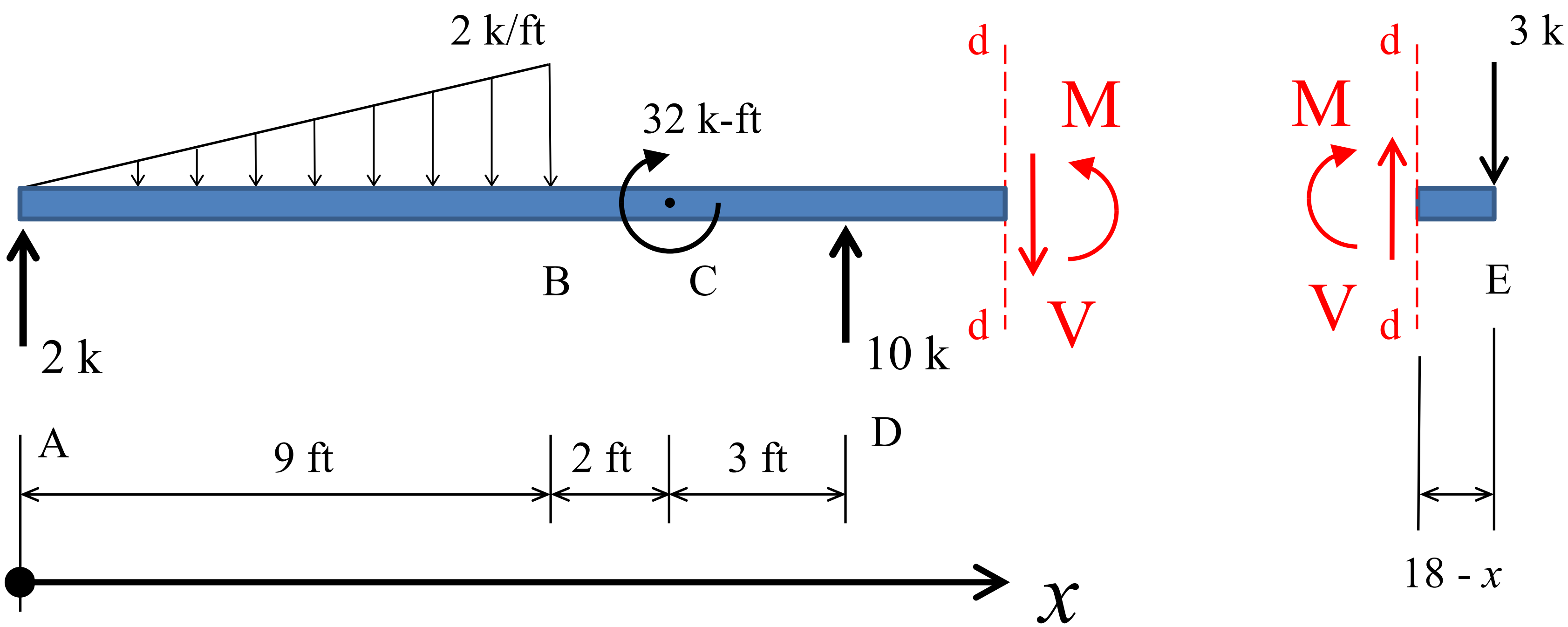
$$\boxed{\frac{dM}{dx} = -7 = V}$$

Plot V and M functions for segment CD



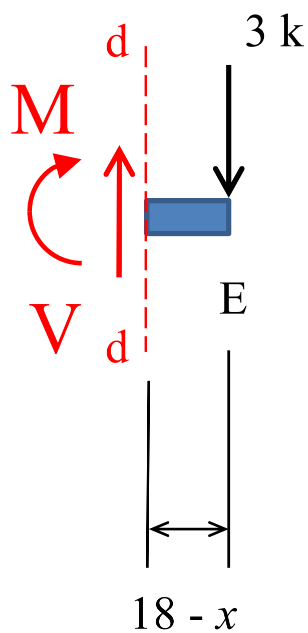
Analysis of Segment DE

FBD of sections on either side of cut at section d-d



FBD of section to the right of cut d-d

Solve for V and M functions for segment CD



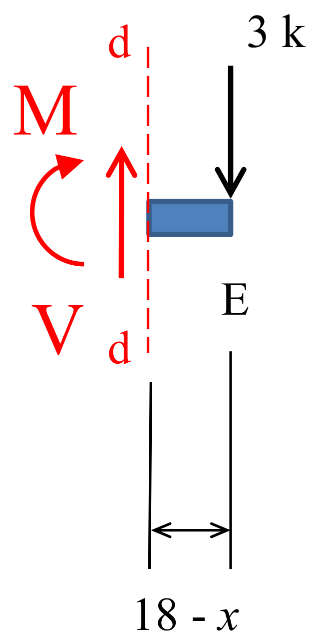
$$+\uparrow \sum F_y = 0$$

$14 \leq x \leq 18$

$V(14) = V_{D+} = 3 \text{ k}$
$\text{V}(18) = V_E = 3 \text{ k}$

$$V = 3$$

Solve for V and M functions for segment DE



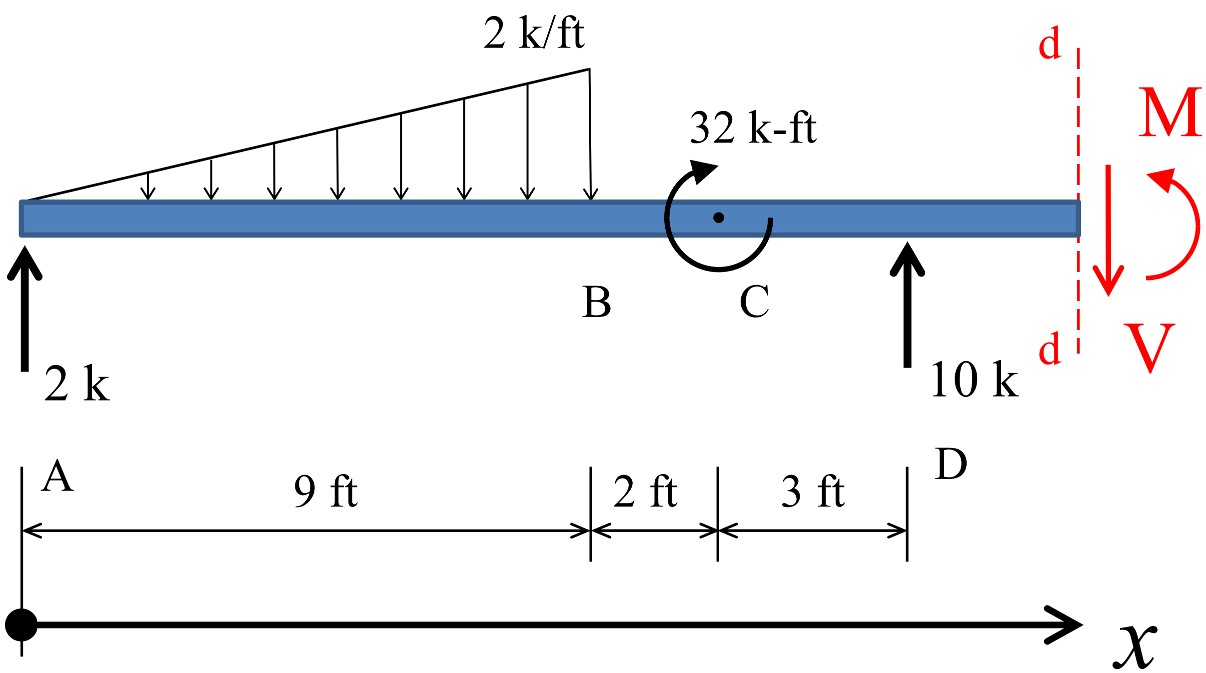
$$\circlearrowleft + \sum M_d = 0$$

$$14 \le x \le 18$$

$$M(14) = M_{D+} = -12 \text{ k-ft}$$
$$M(18) = M_E = 0$$

$$M = -54 + 3x$$

Relationships between w, V, and M



$$w = 0$$

$$V = 3$$

$$M = -54 + 3x$$

$$\boxed{\frac{dV}{dx} = 0 = w}$$

$$\boxed{\frac{dM}{dx} = 3 = V}$$

Plot V and M functions for segment DE