

Equilibrium of Rigid Bodies and Construction Free-Body Diagrams

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General Procedure for the Analysis of Bodies in Static Equilibrium

- Choose the free body to isolate;
- Draw a **Free Body Diagram (FBD)** of the body;
 - Isolate the body from all of its surroundings,
 - Magnitudes and directions of all known and unknown forces acting on the body should be included and clearly indicated,
 - Indicate dimensions on the FBD,
- Write the **equations of equilibrium** and solve the equations for the unknown quantities.

General Procedure for the Construction of Free Body Diagrams

- Choose the free body to isolate;
- Isolate the body from all of its surroundings;
- Magnitudes and directions of all known and unknown forces acting on the body should be included and clearly indicated;
- Dimensions should be indicated on the FBD.

Most errors in mechanics problems result from a mistake in the FBD

Reactive Forces at Supports for Planar Structures

Reactive force with unknown magnitude and known line of action

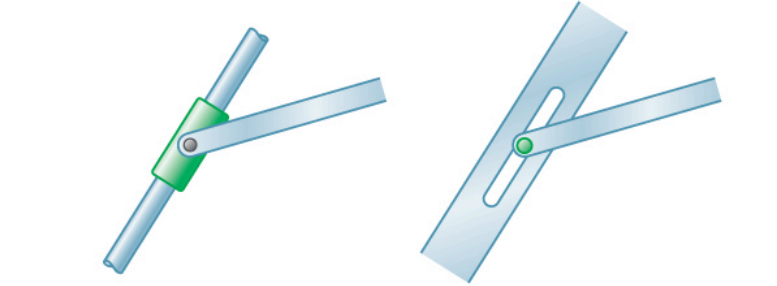
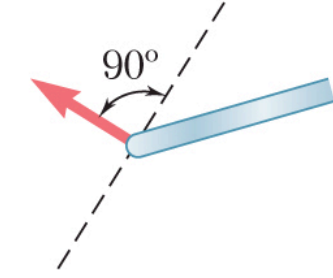
Support or Connection	Reaction	Number of Unknowns
<p>Rollers Rocker Frictionless surface</p>	<p>Force with known line of action</p>	1

force is perpendicular to surface

Example of a Roller Support

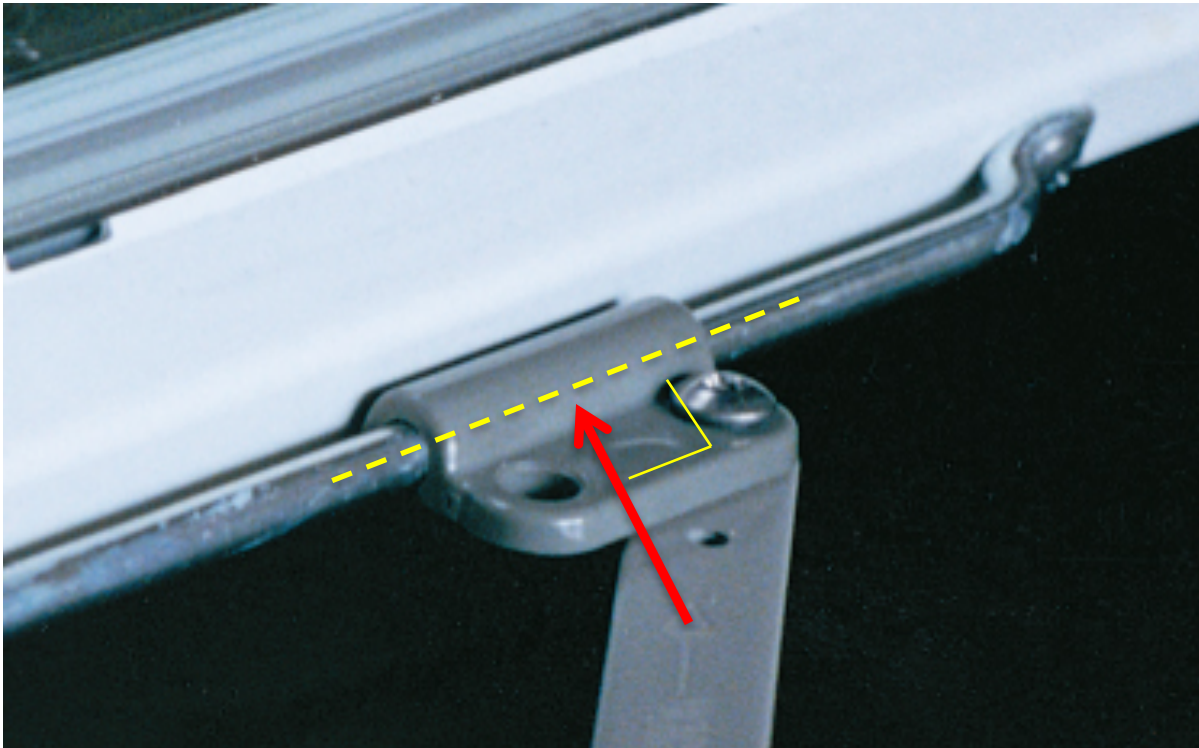


Reactive force with unknown magnitude and known line of action

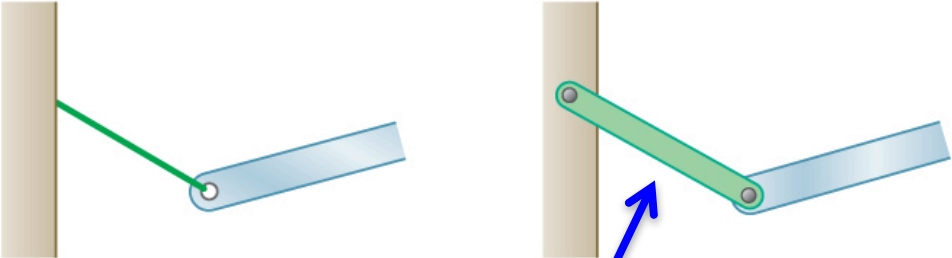

Support or Connection	Reaction	Number of Unknowns
 <p style="display: flex; justify-content: space-around;"> Collar on frictionless rod Frictionless pin in slot </p>	 <p style="text-align: center;">Force with known line of action</p>	1

Force is perpendicular to rod or slot

Example of a Collar on a Guide Rod



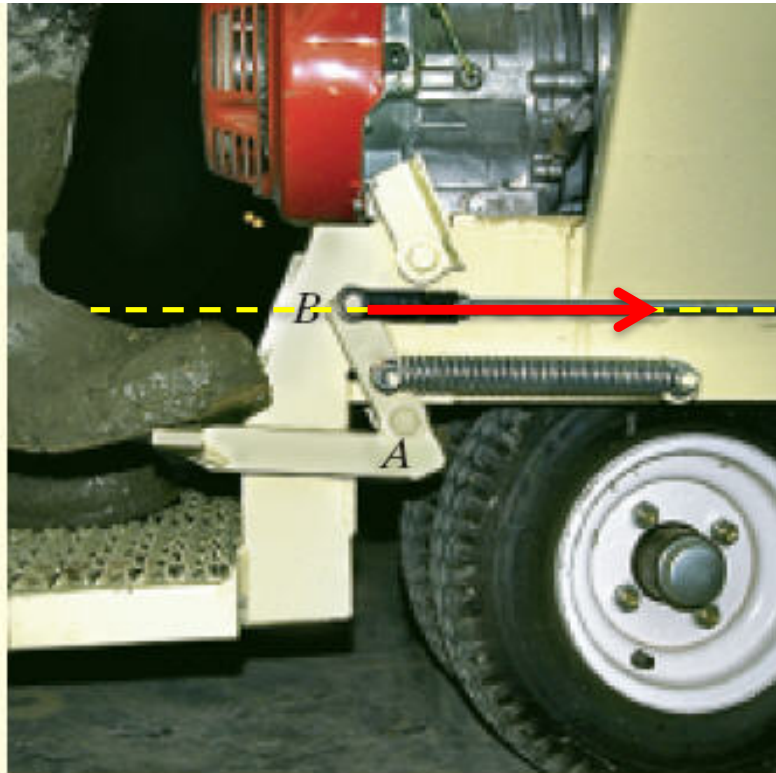
Reactive force with unknown magnitude and known line of action

Support or Connection	Reaction	Number of Unknowns
 <p data-bbox="310 878 537 922">Short cable</p> <p data-bbox="863 878 1056 922">Short link</p>	 <p data-bbox="1289 883 1650 971">Force with known line of action</p>	<p data-bbox="1818 781 1839 824">1</p>



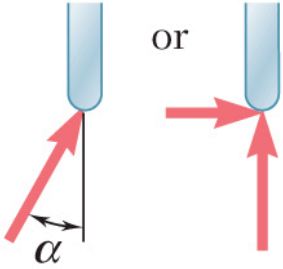
Link is a two-force member

Force is directed along the line of the cable or link

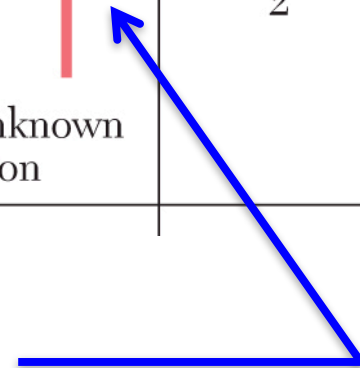
Examples of Short Links



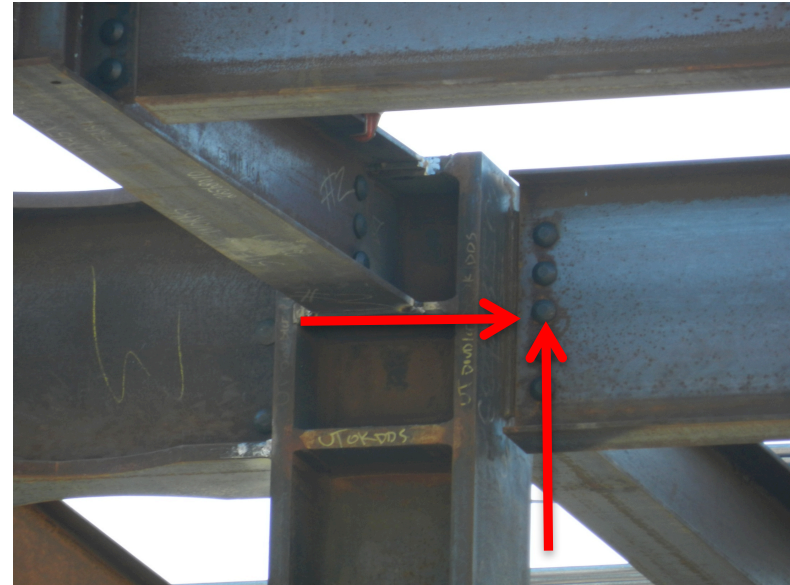
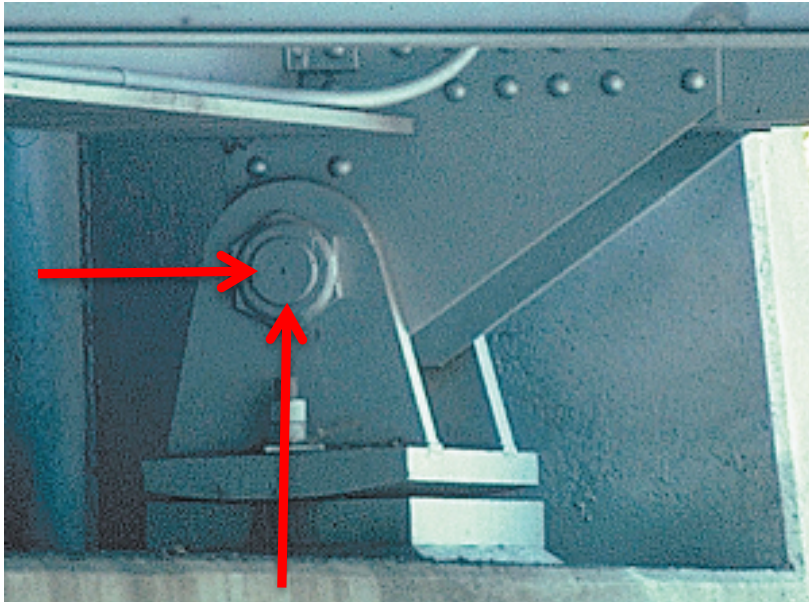
Reactive force with unknown magnitude and unknown direction

Support or Connection	Reaction	Number of Unknowns
<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p data-bbox="327 992 606 1078">Frictionless pin or hinge</p> </div> <div style="text-align: center;">  <p data-bbox="816 992 1075 1036">Rough surface</p> </div> </div>	<div style="display: flex; justify-content: center; align-items: center;">  <p data-bbox="1224 992 1549 1078">Force of unknown direction</p> </div>	2

Usually the most convenient way
to express the two unknowns

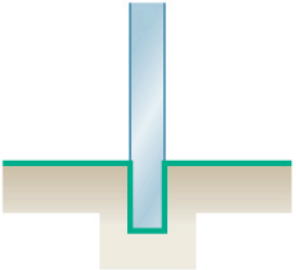
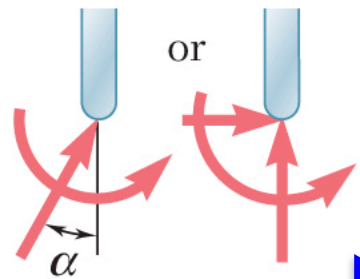


Examples Pin Supports and Pin Connections

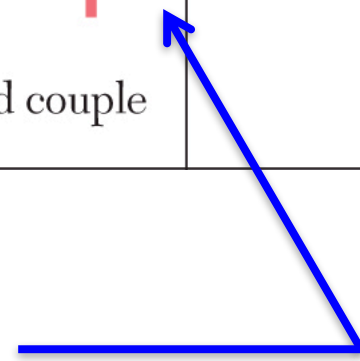


This beam connection where only the beam web is bolted to the column is usually modeled as a pin connection

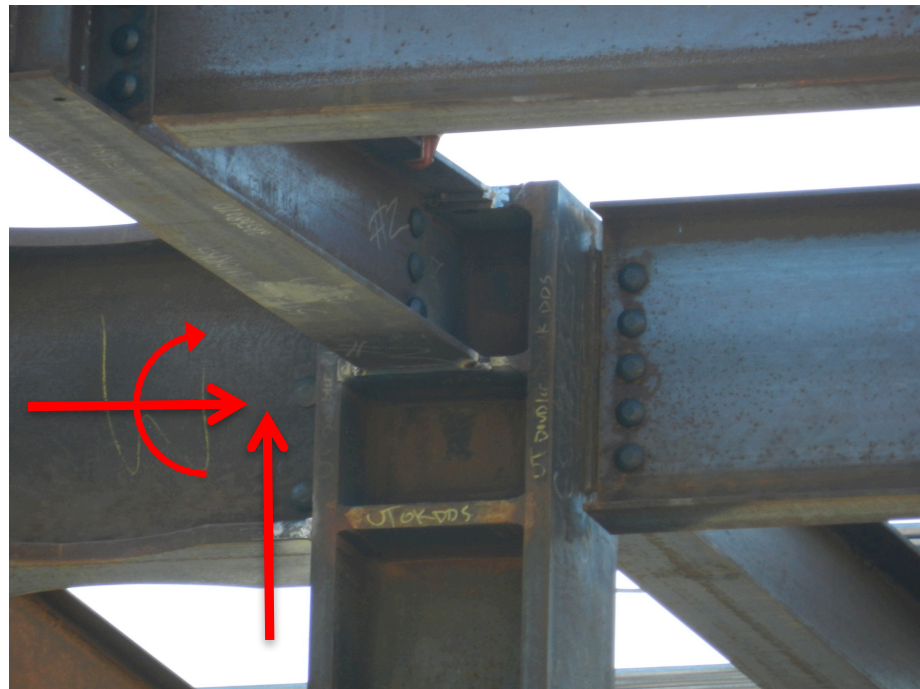
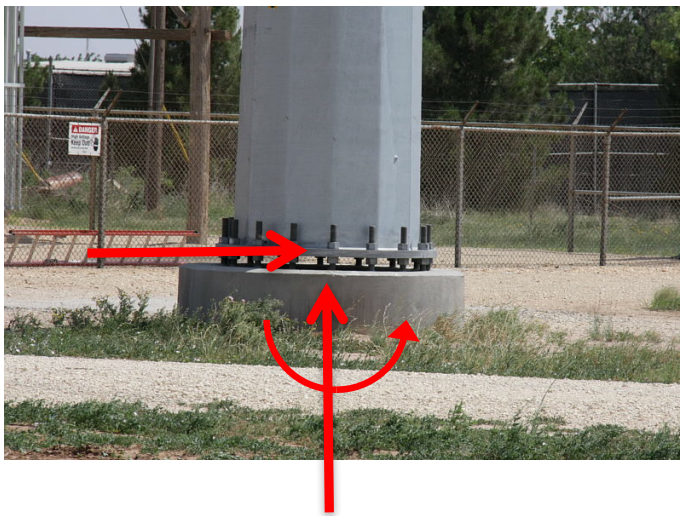
Reactive force with unknown magnitude, unknown direction, and an unknown moment

Support or Connection	Reaction	Number of Unknowns
 <p>Fixed support</p>	 <p>Force and couple</p>	3

Usually the most convenient way to express the three unknowns



Examples Rigid Supports and Rigid Connections



This beam connection where the beam flanges are welded to the column is usually modeled as a rigid (moment resisting) connection

Scalar Equations of Static equilibrium

General three-dimensional
body

$$\begin{aligned}\sum F_x = 0 & \quad \sum F_y = 0 & \quad \sum F_z = 0 \\ \sum M_x = 0 & \quad \sum M_y = 0 & \quad \sum M_z = 0\end{aligned}$$

General two-dimensional
(planar) body

$$\begin{aligned}\sum F_x = 0 & \quad \sum F_y = 0 & \quad \sum F_z = 0 \\ \sum M_x = 0 & \quad \sum M_y = 0 & \quad \sum M_z = 0\end{aligned}$$

Scalar Equations of Static Equilibrium for Concurrent Force Systems

Three-dimensional
body with concurrent forces

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

Two-dimensional (planar)
body with concurrent
forces

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$