Constructing Beam Influence Lines Using the Muller-Breslau Principle

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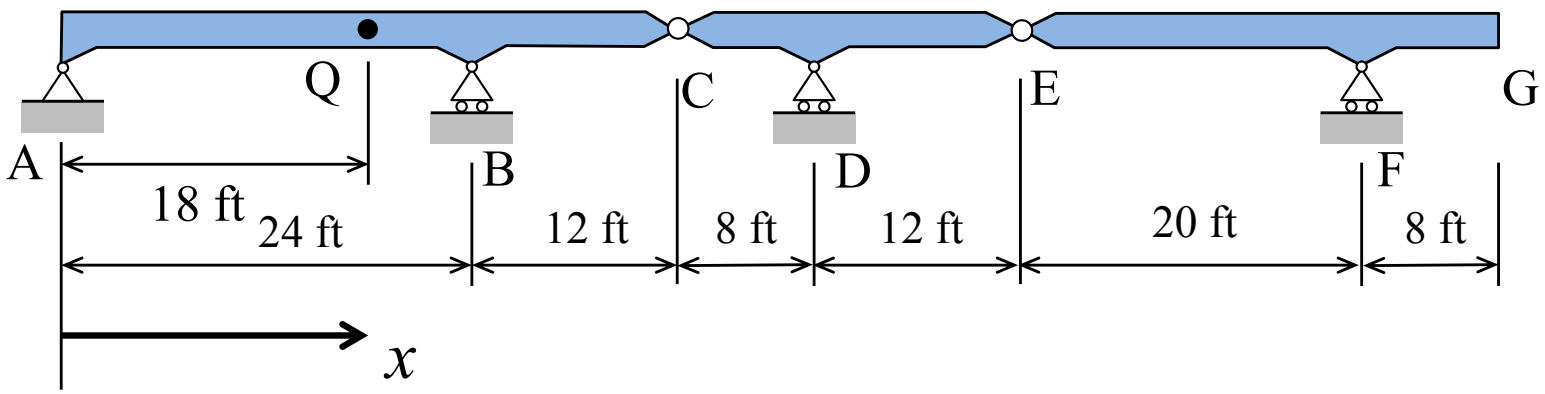
Example Problem

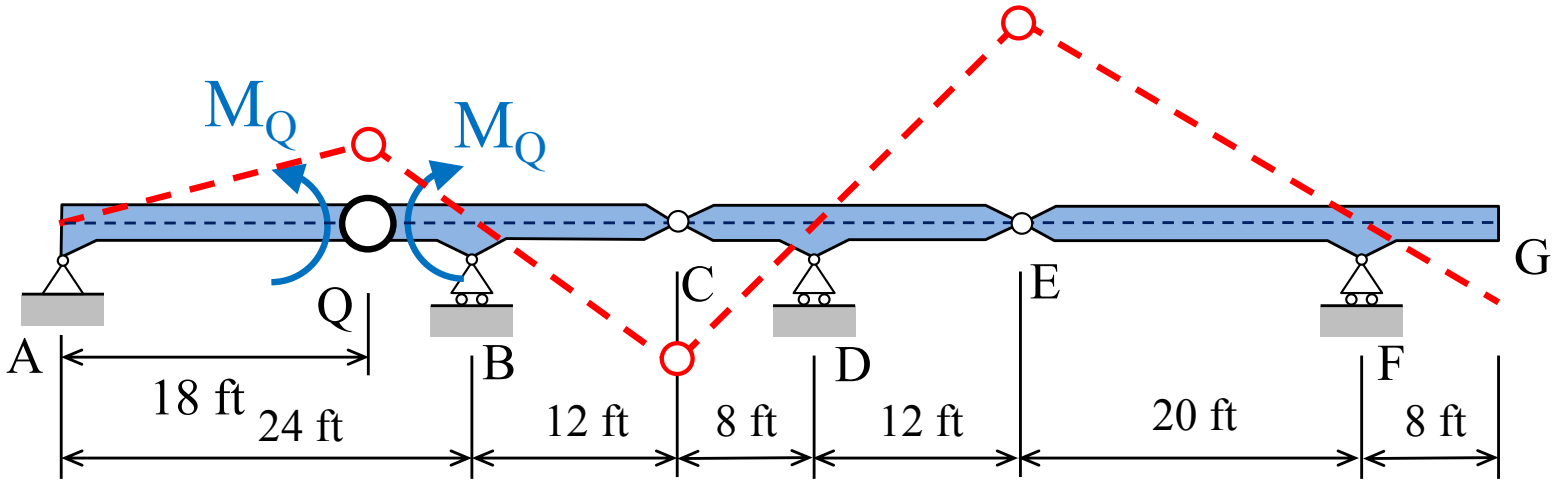


A beam is pin supported at point A and roller supported at points B, D, and F. The beam contains internal hinges at points C and E.

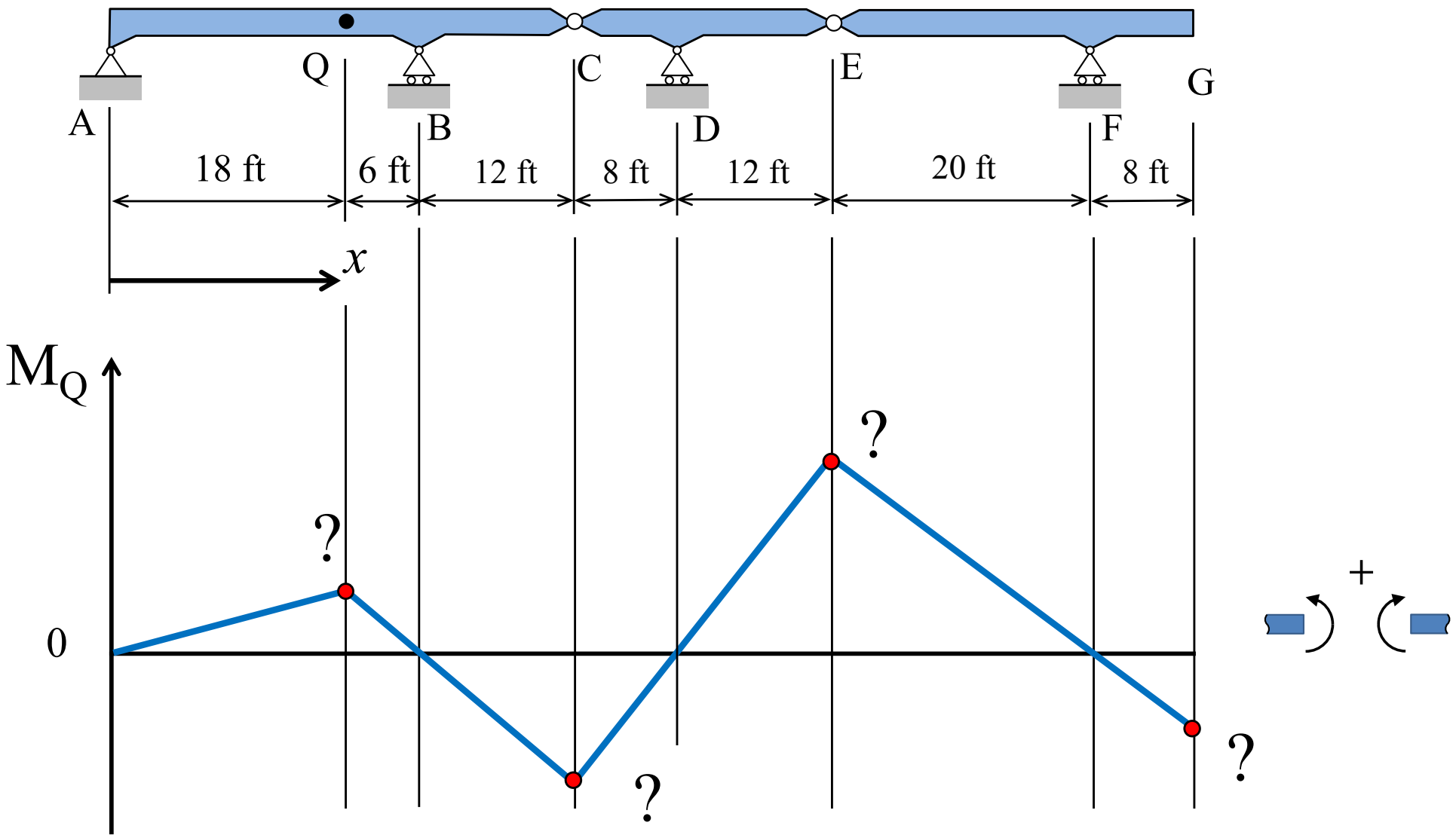
Construct the influence line for the bending moment at point Q.

Muller-Breslau Principle to Find the Shape of the M_Q Influence Line

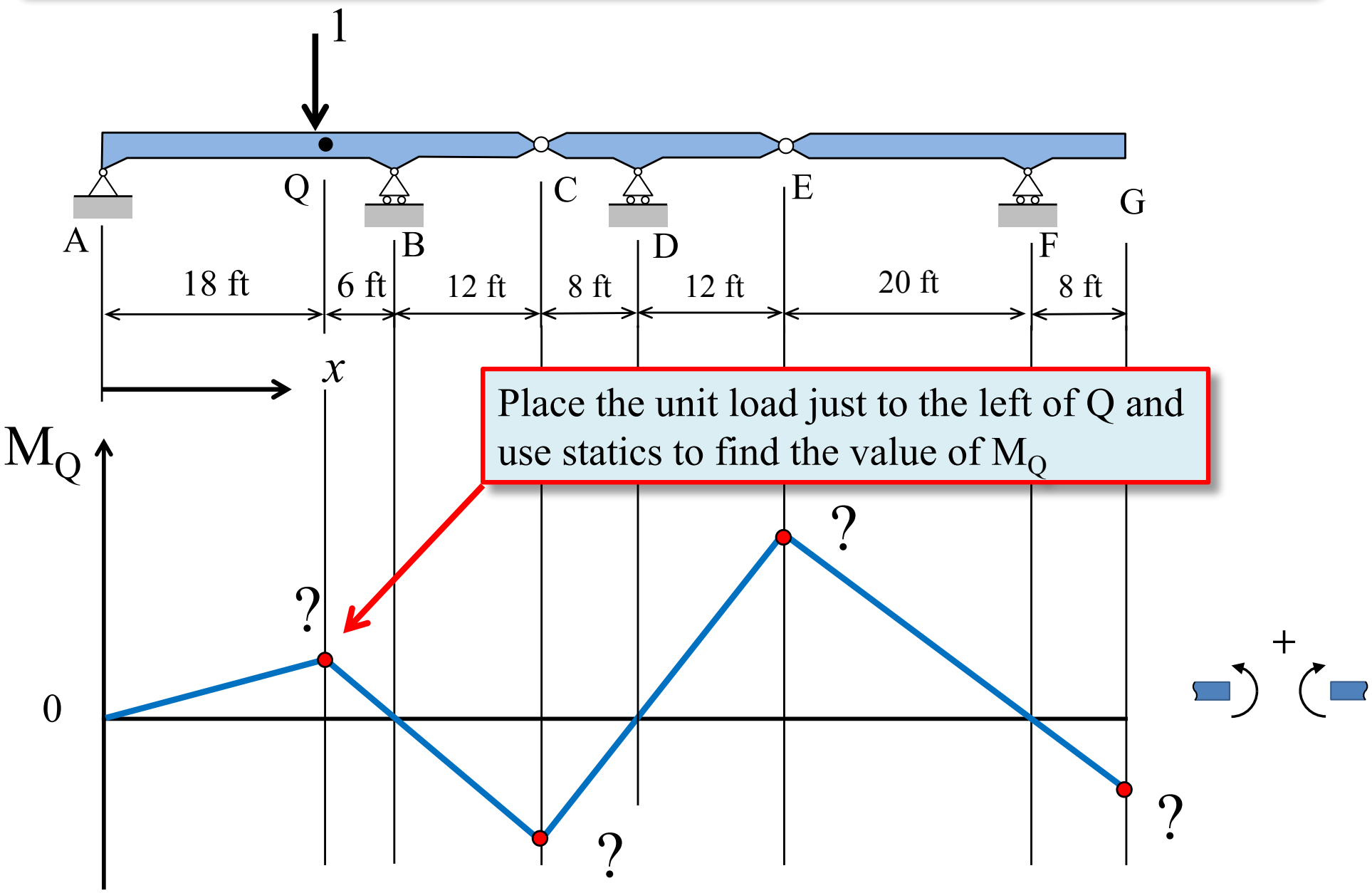
1. Remove the ability of the beam to resist bending moment at point Q. This is the modified unstable structure;
2. Apply the response quantity, M_Q, consistent with the chosen positive sign convention;
3. The rigid body motion of the modified structure is the shape of the M_Q influence line.



Shape of M_Q Influence Line

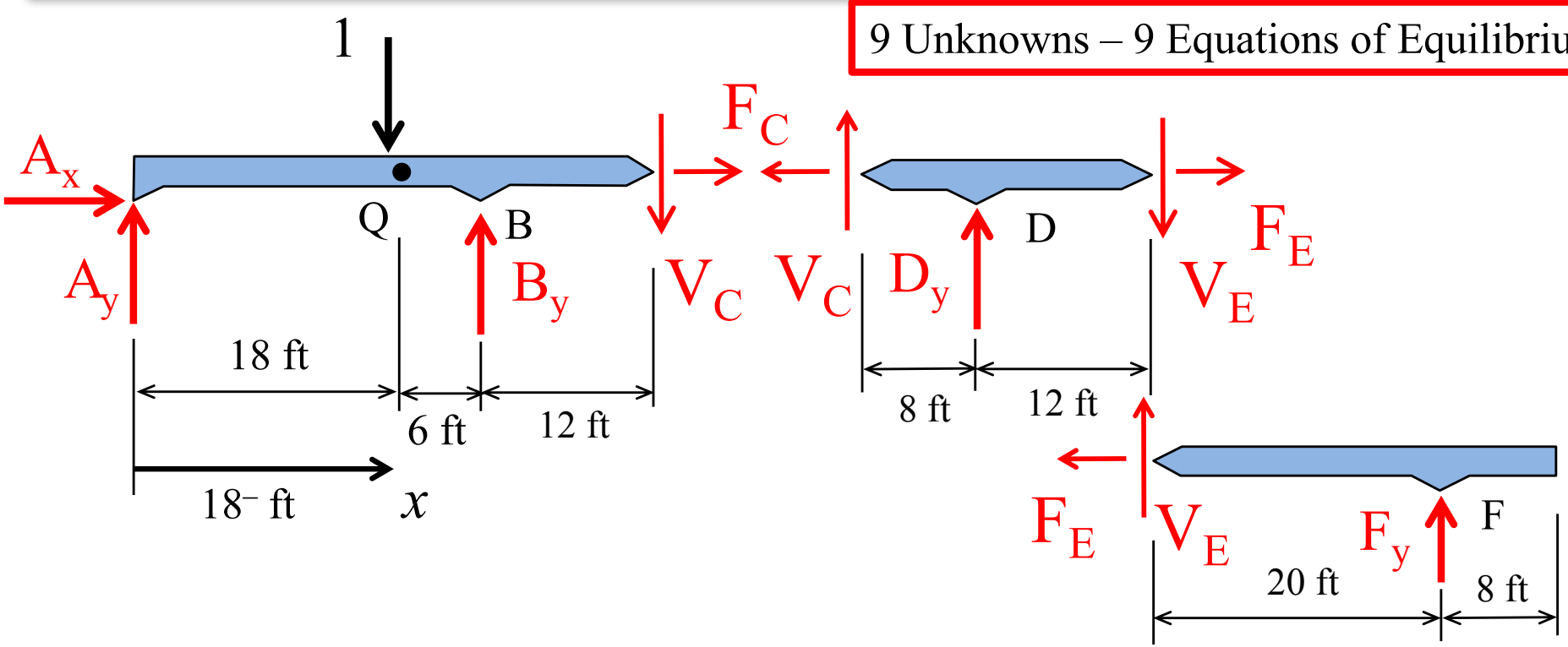


Use Shape of Influence Line to Place Unit Load



Place Unit Load at $x = 18^-$ ft (Just to the Left of Point Q)

9 Unknowns – 9 Equations of Equilibrium



$\circlearrowleft + \sum M_A = 0 \rightarrow B_y = 0.75$

$\rightarrow + \sum F_x = 0 \rightarrow A_x = 0$

$+\uparrow \sum F_y = 0 \rightarrow A_y = 0.25$

$D_y = 0$

$F_C = 0$

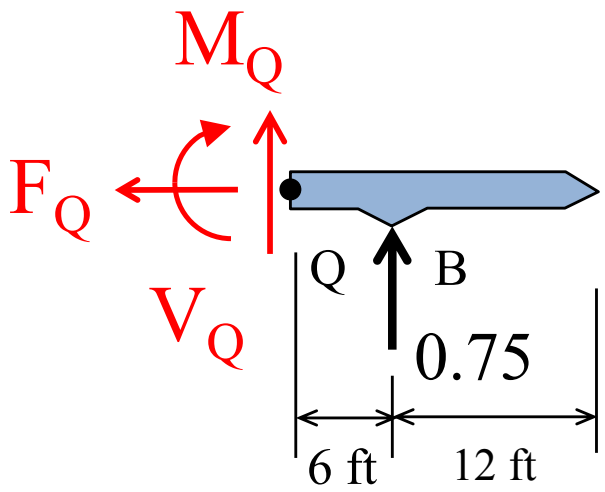
$V_C = 0$

$F_y = 0$

$F_E = 0$

$V_E = 0$

FBD of Segment QBC for Unit Load at $x = 18^-$ ft (Just to the Left of Point Q)



$V_C = 0$

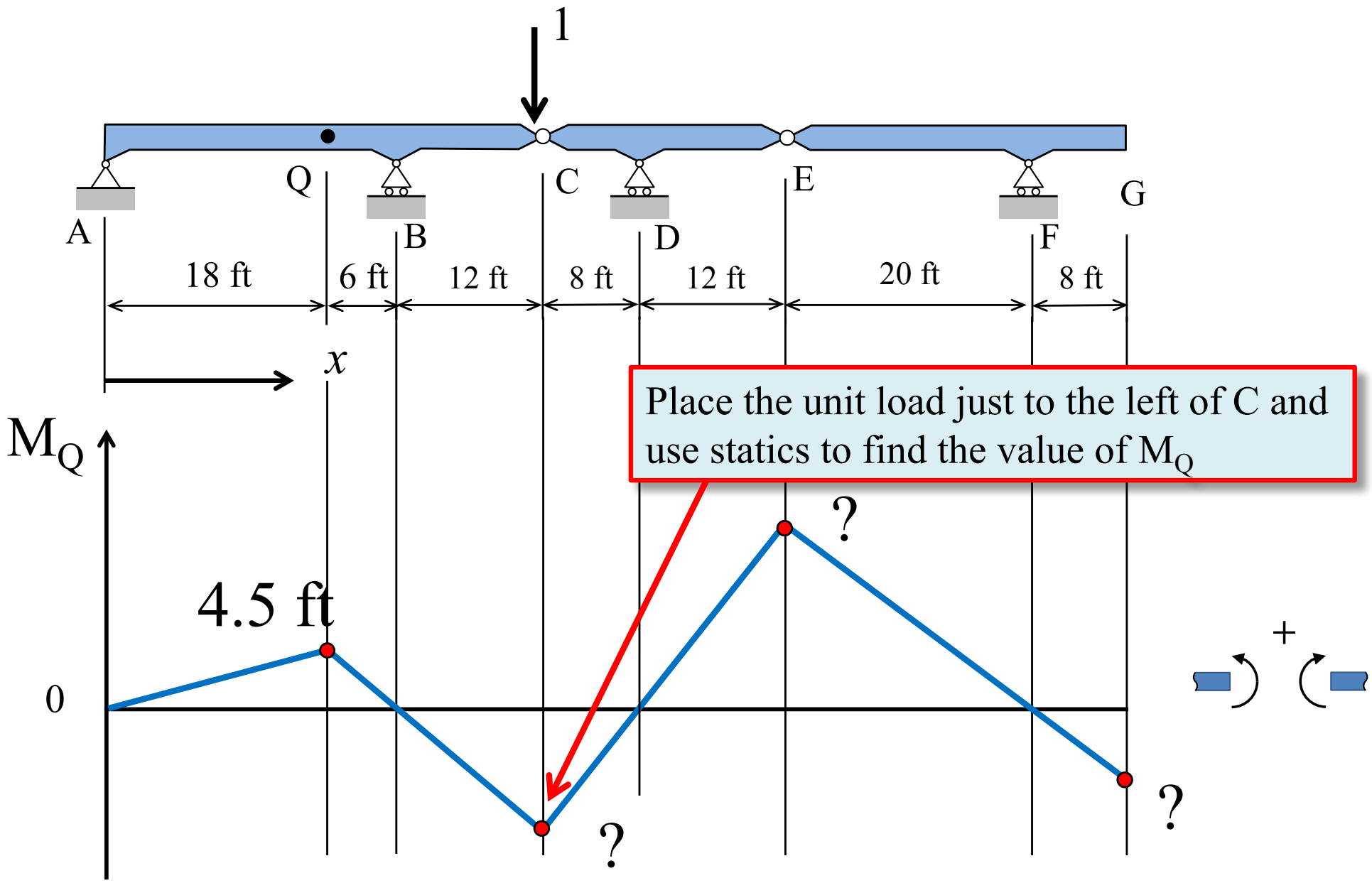
$F_C = 0$

$$\circlearrowleft + \sum M_Q = 0 \rightarrow \boxed{M_Q = 4.5 \text{ ft}}$$

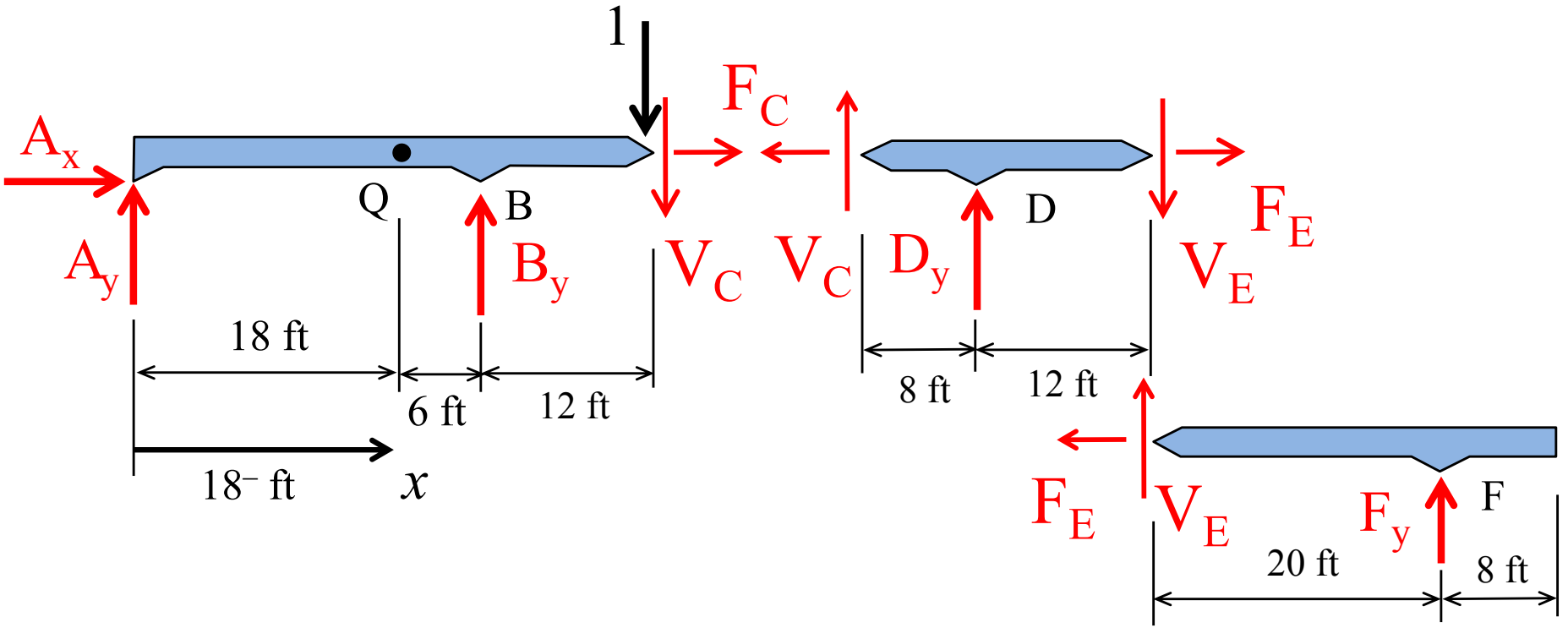
$$\rightarrow + \sum F_x = 0 \rightarrow F_Q = 0$$

$$+\uparrow \sum F_y = 0 \rightarrow V_Q = -0.75$$

Use Shape of Influence Line to Place Unit Load



Place Unit Load at $x = 36^-$ ft (Just to the Left of Point C)



$\circlearrowleft + \sum M_A = 0 \rightarrow B_y = 1.5$

$+\rightarrow \sum F_x = 0 \rightarrow A_x = 0$

$+\uparrow \sum F_y = 0 \rightarrow A_y = -0.5$

$D_y = 0$

$F_C = 0$

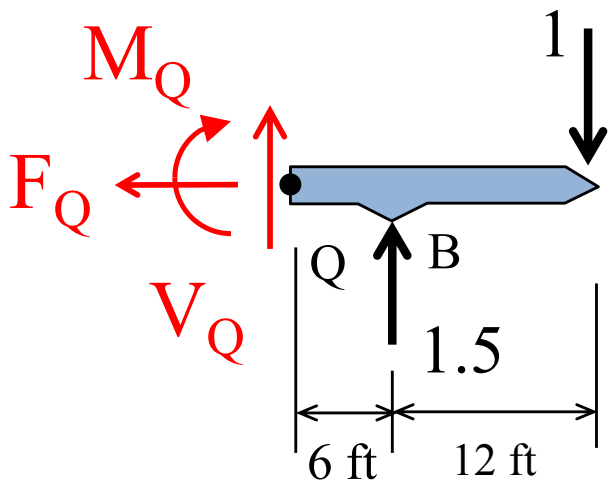
$V_C = 0$

$F_y = 0$

$F_E = 0$

$V_E = 0$

FBD of Segment QBC for Unit Load at $x = 36^-$ ft (Just to the Left of Point C)



$V_C = 0$

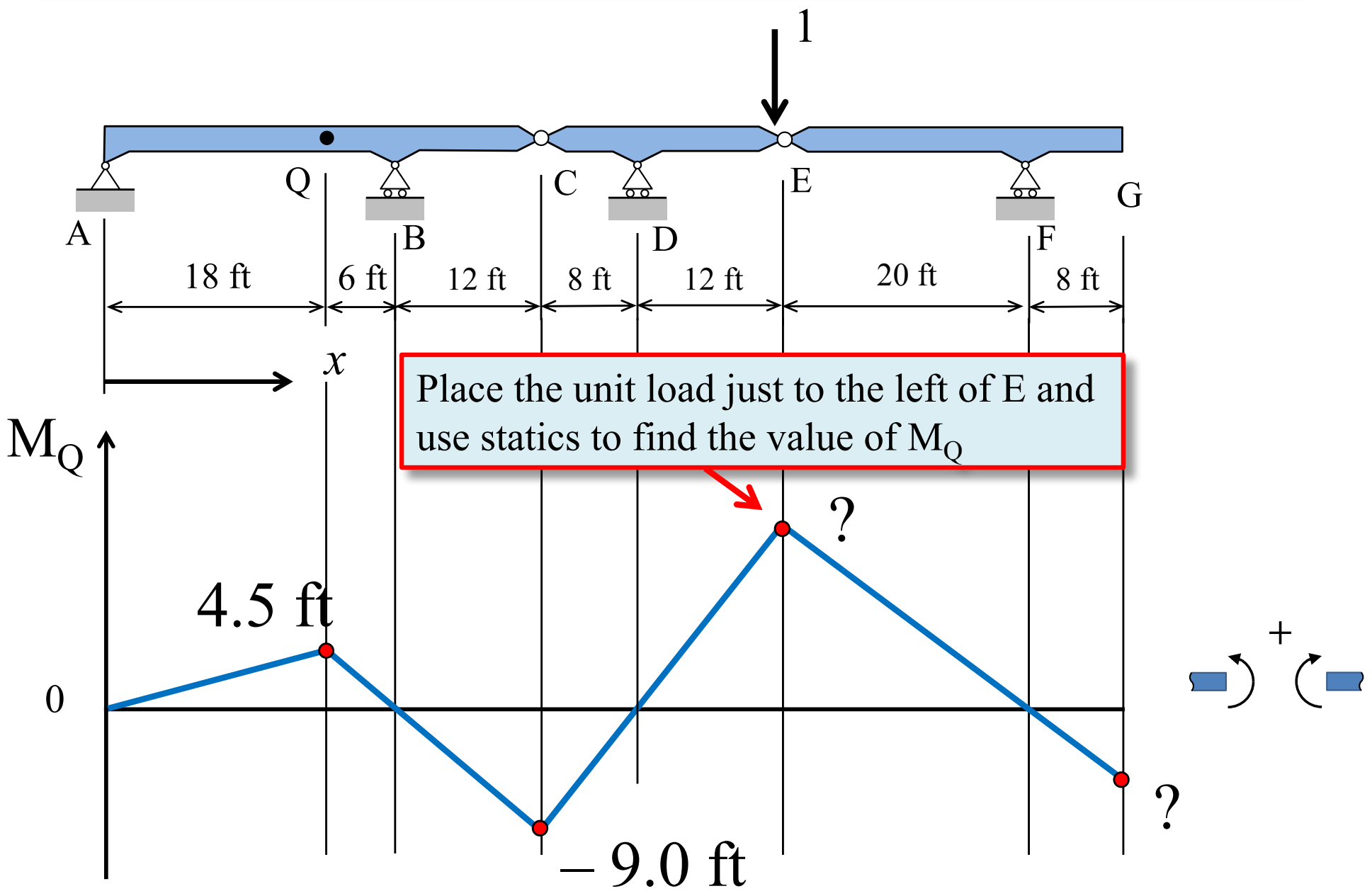
$F_C = 0$

$\circlearrowleft + \sum M_Q = 0 \rightarrow$ $M_Q = -9.0$ ft

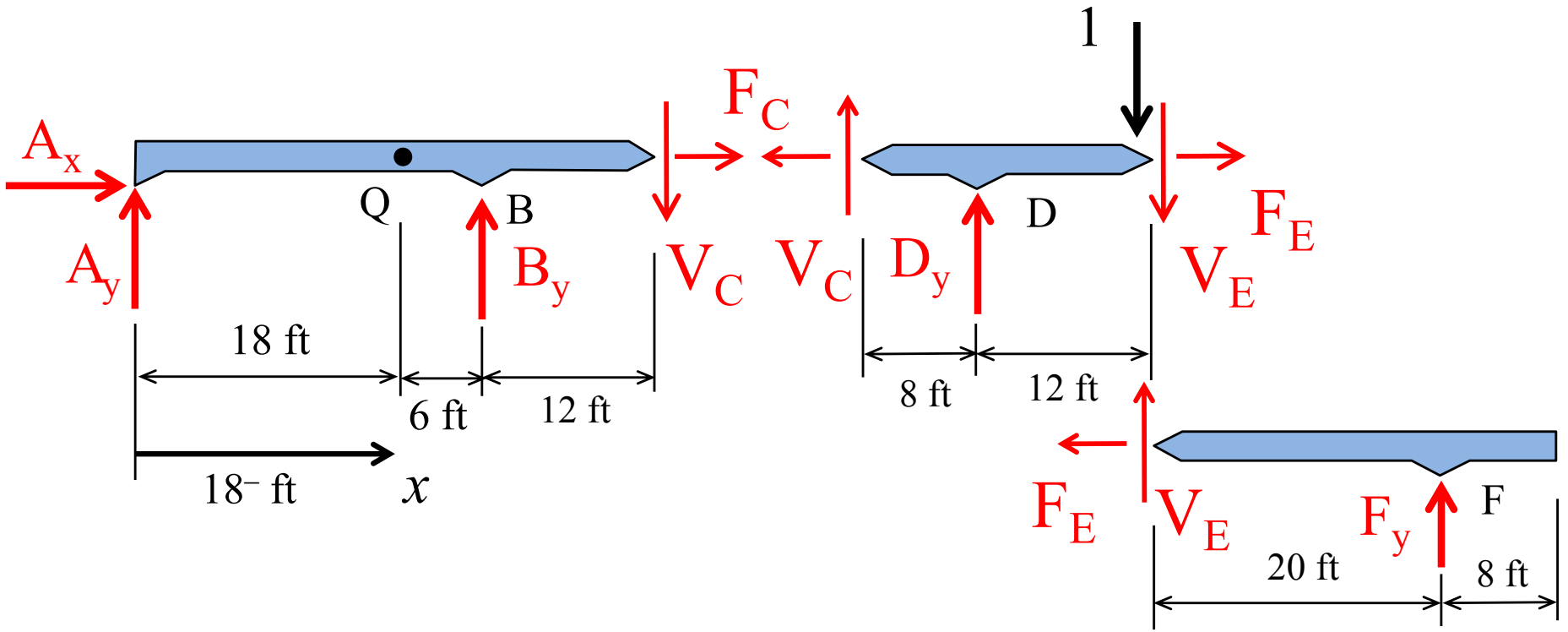
$\rightarrow + \sum F_x = 0 \rightarrow F_Q = 0$

$+\uparrow \sum F_y = 0 \rightarrow V_Q = -0.5$

Use Shape of Influence Line to Place Unit Load



Place Unit Load at $x = 56^-$ ft (Just to the Left of Point E)



$\circlearrowleft + \sum M_A = 0 \rightarrow B_y = -2.25$

$\pm \rightarrow \sum F_x = 0 \rightarrow A_x = 0$

$+\uparrow \sum F_y = 0 \rightarrow A_y = 0.75$

$D_y = 2.5$

$F_C = 0$

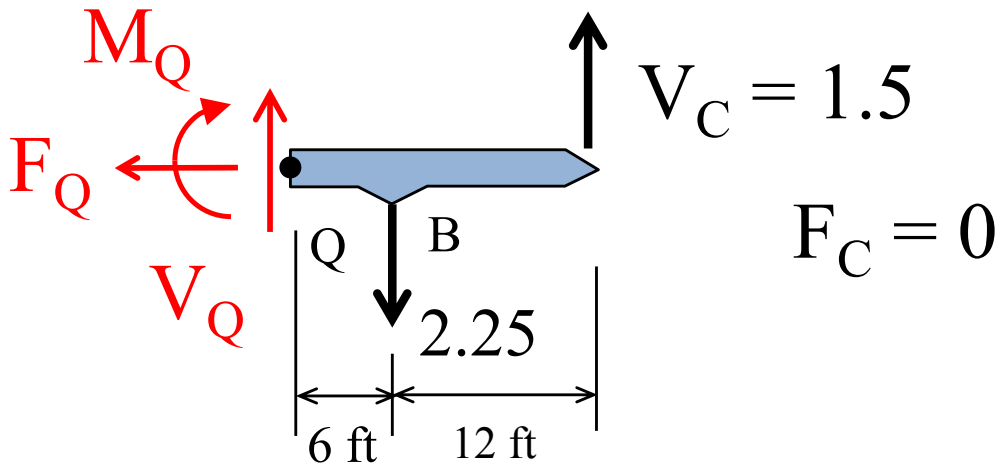
$V_C = -1.5$

$F_y = 0$

$F_E = 0$

$V_E = 0$

FBD of Segment QBC for Unit Load at $x = 56^-$ ft (Just to the Left of Point C)

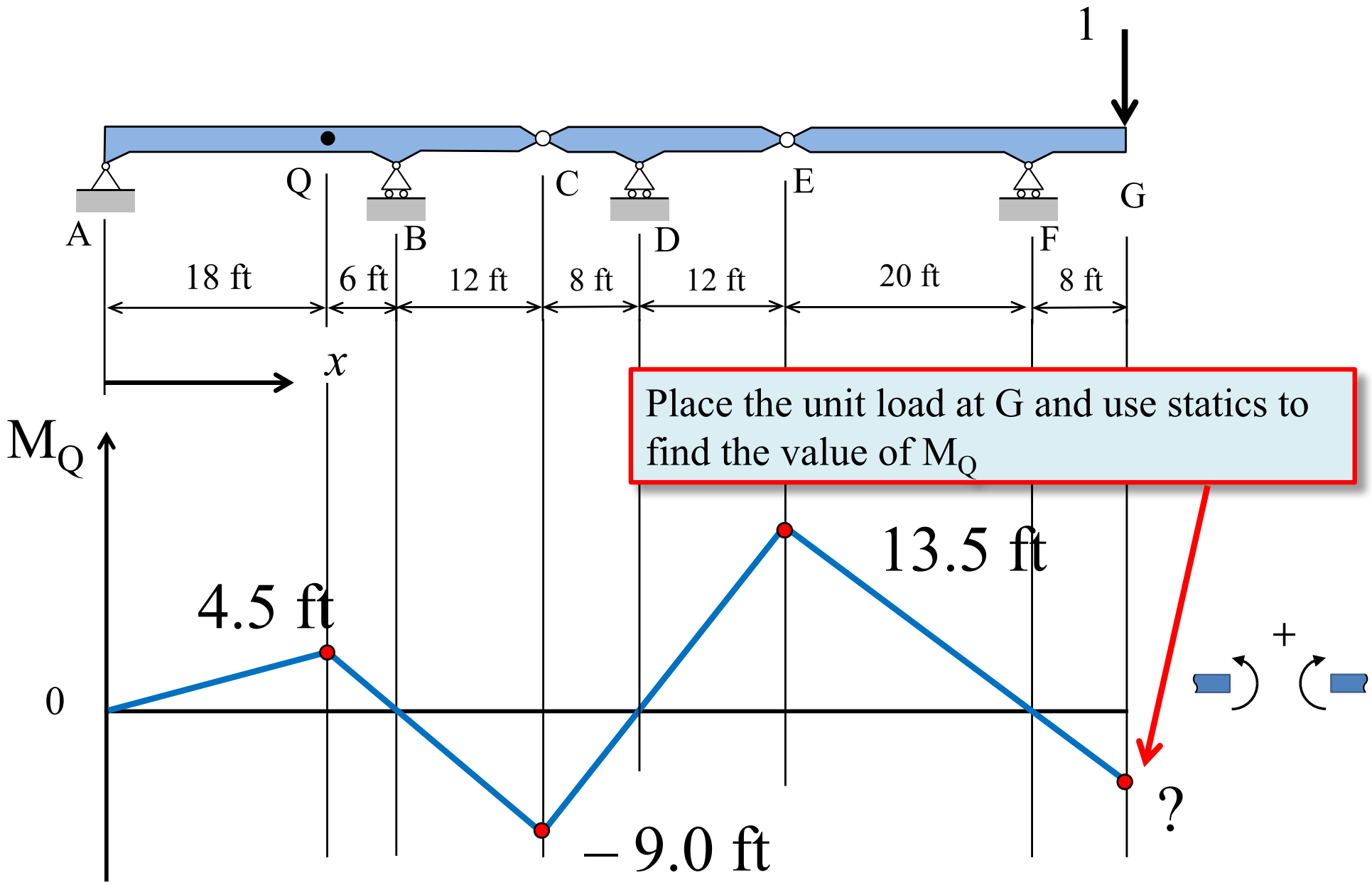


$$\circlearrowleft + \sum M_Q = 0 \rightarrow \boxed{M_Q = 13.5 \text{ ft}}$$

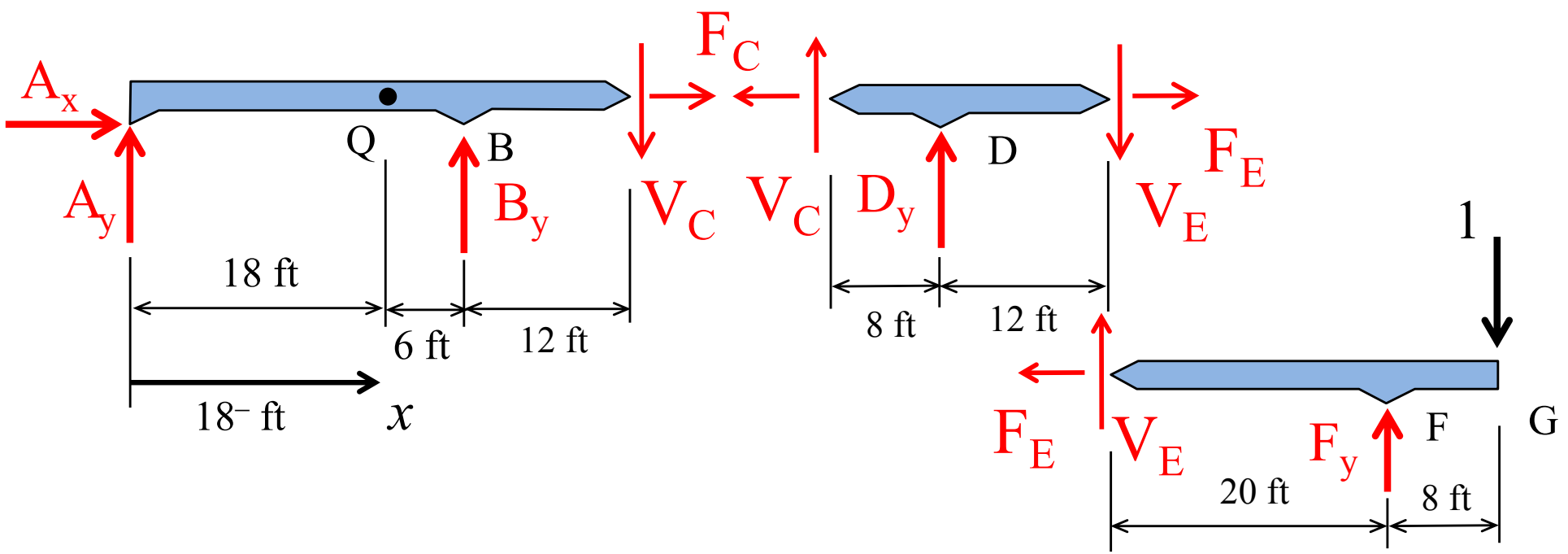
$$\rightarrow + \sum F_x = 0 \rightarrow F_Q = 0$$

$$+\uparrow \sum F_y = 0 \rightarrow V_Q = 1.25$$

Use Shape of Influence Line to Place Unit Load



Place Unit Load at $x = 84$ ft (At Point G)



$\circlearrowleft + \sum M_A = 0 \rightarrow B_y = 0.9$

$\rightarrow + \sum F_x = 0 \rightarrow A_x = 0$

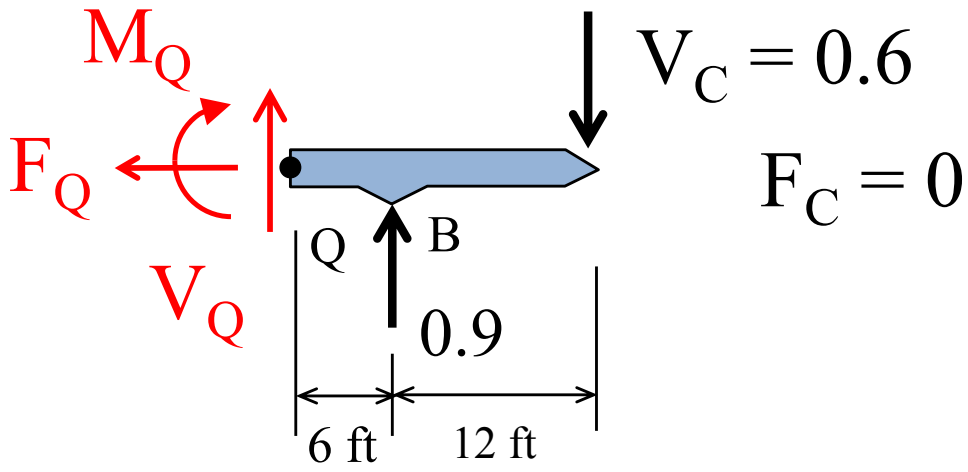
$+\uparrow \sum F_y = 0 \rightarrow A_y = -0.3$

$D_y = -1.0$ $\qquad$ $F_y = 1.4$

$F_C = 0$ $\qquad$ $F_E = 0$

$V_C = 0.6$ $\qquad$ $V_E = -0.4$

FBD of Segment QBC for Unit Load at $x = 84$ ft (At Point G)

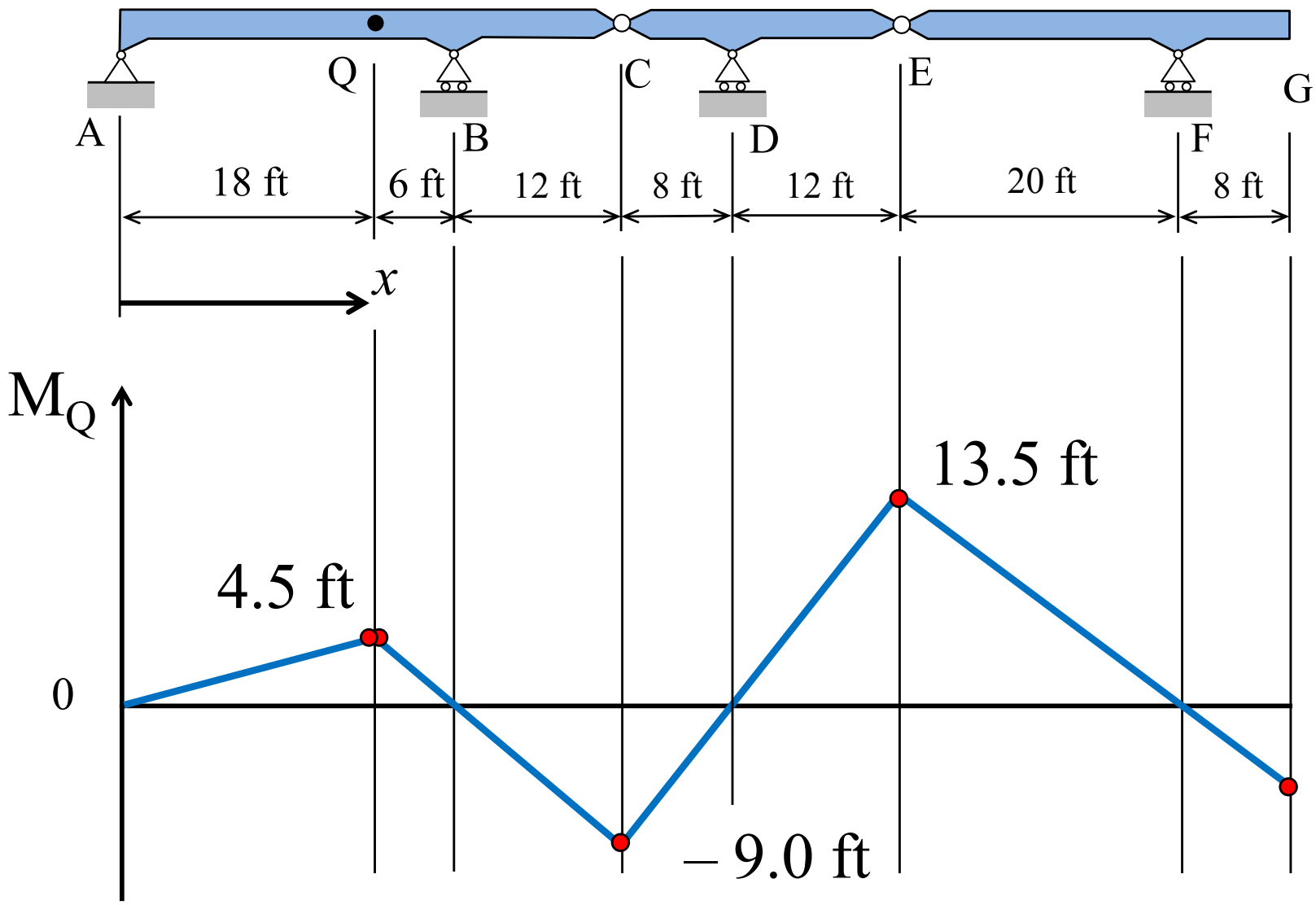


$$\circlearrowleft + \sum M_Q = 0 \rightarrow \boxed{M_Q = -5.4 \text{ ft}}$$

$$\rightarrow + \sum F_x = 0 \rightarrow F_Q = 0$$

$$+\uparrow \sum F_y = 0 \rightarrow V_Q = -0.3$$

M_Q Influence Line



x =	M_Q
0	0
18^- ft	4.5 ft
18^+ ft	4.5 ft
24 ft	0
36^- ft	– 9.0 ft
44 ft	0
56^- ft	13.5 ft
76 ft	0
84 ft	– 5.4 ft