Characterization of High-Q Resonators for Microwave-Filter Applications

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***Abstract*—A one-port reflection technique is developed to measure the unloaded Q and external Q of a microwave resonator. The unique procedure of measuring unloaded Q is outlined in three easy steps. A sample chart is provided to further simplify the process. This method is so simple that even a scalar network analyzer is adequate for the measurement. In addition, a time-delay response around the resonator resonant frequency is also derived and presented. This theoretical result, combined with the advanced capability of modern vector network analyzers, has been proven to be very useful for characterization and tuning of the external Q of a resonator. All the results derived are verified by practical measurement. Finally, this technique is applied to the realization and tuning of a six-pole dielectric loaded cavity filter.**

I. Introduction

Microwave resonators are building blocks [1], [2] for many microwave devices, such as filters, multiplexers, and oscillators. The resonator unloaded Q (Q_u) and external Q (Q_e) are two fundamental parameters in microwave resonator designs. The former is an important index to determine the limitation of applications, and the latter dictates how the resonator interacts with other microwave devices in the system.

The resonator Q_u measurement has always been an attractive research topic. Excellent literature [3]–[14] is available for most microwave applications. However, some of these methods require sophisticated mathematical treatment or complicated procedures. In this paper, based on a well-known equivalent circuit, we present an original expression for the resonator Q_u as a function of the generalized one-port loaded Q response. Simple procedures and charts are developed and can be directly applied to experiments using either a scalar or vector network analyzer. The advantage of the one-port reflection measurement over the two-port transmission type for resonators used in microwave filters is that the Q_u can usually be measured in the same filter housing so that all the packaging effects are included. It is also more cost effective since it does not require additional fabrication.

The input/output coupling design of a microwave filter is somewhat similar to the resonator Q_u characterization. In this case, the coupling or, equivalently, the Q_e instead of the Q_u, is the parameter which not only should be precisely measured, but also should be adjusted to match the required value for a specific filter response. In any case, one can measure the phase or time delay of the input reflection coefficient to evaluate the input/output coupling. Using the same equivalent circuit, a closed-form relationship between the one-port time delay and input coupling (or Q_e) is also presented. By measuring the phase and delay of the reflection coefficient simultaneously, the measurement reference plane can be properly identified and, thus, the measurement accuracy is guaranteed.

II. Equivalent Circuit for a Coupled Resonator

Fig. 1 shows an equivalent circuit for a series resonator coupled to a source impedance Z_o. The definition and realization of the impedance (K) inverter can be found in [1] and a generalized concept of the inverters has been suggested by Levy [15]. Alternatively, the inverters can be directly derived from the scattering matrix of a lossless reciprocity of microwave junction [16]. Due to the duality nature of series and parallel resonators, only the former case will be discussed without loss of generality. As shown in Fig. 1, the input resistance presented to the series resonator is defined as r_A and is given by $r_A = K_{01}^2/Z_o$, and the reflection coefficient from this resistance looking into the resonator is

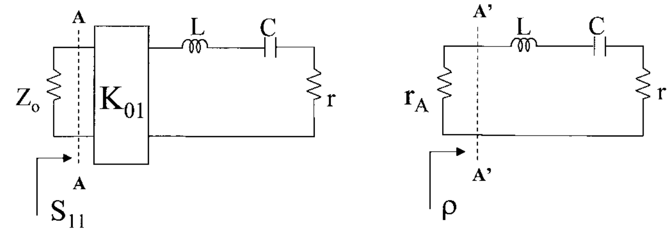

Fig. 1. Equivalent circuit of a series resonator.

$$\rho = \frac{(r - r_A) + j(\omega L - 1/\omega C)}{(r + r_A) + j(\omega L - 1/\omega C)} = \frac{(1-\beta) + jQ_u\Omega}{(1+\beta) + jQ_u\Omega} \tag{1}$$

where $Q_u = \frac{\omega_o L}{r}$, $\Omega = \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}$, $\omega_o = \frac{1}{\sqrt{LC}}$, and β is the coupling parameter defined as r_A/r.

It was pointed out in [1] that ρ is differed from the actual measurable reflection coefficient S_{11} by a phase difference of 180°. This phase difference can be regarded as a shifting of the reference plane in measurement and is not important for characterizing the resonator. Mathematically, $S_{11} = \rho \exp\{-j\pi\}$ and the measured return loss is $\mathrm{RL} = -20\log|S_{11}| = -20\log|\rho|$. Here, we have adopted the convention that RL is a positive real number in decibels.

III. Unload Q Measurement

For the following discussions, a generalized loaded Q is introduced as $Q_L(x,\beta) \equiv \omega_o/(\Delta\omega)_x$, where $(\Delta\omega)_x$ is the bandwidth measured at the $-x$-dB points of the input return loss, as illustrated in Fig. 2. Most modern network analyzers (such as the HP8753C) can measure $Q_L(x,\beta)$ automatically for any chosen value of x. This measurable generalized $Q_L(x,\beta)$ is a function of both x and β, whereas the conventional loaded Q is independent of x and defined by the familiar expression of $\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_u}$, where $Q_u = \omega_o L/r$ and $Q_e = \omega_o L/r_A$. Until now, the Q_u measurement using the reflection-coefficient technique requires the following sequence [3], [4]:

1) measure the return loss at resonant frequency;
2) determine the coupling condition (under- or over-coupled);
3) calculate the coupling parameter β;
4) calculate the correct return loss level ρ_x in which Δf is to be measured;
5) go back to the network analyzer and measure the bandwidth Δf;
6) evaluate Q_L and then calculate Q_u using $Q_u = Q_L(1+\beta)$.

By introducing the generalized loaded $Q_L(x,\beta)$, the measurement can be easily mapped into the Q_u for any value of β with no prior calculation. Derivation of such a technique is presented below.

Around the resonance ω_o, $\Omega = \omega/\omega_o - \omega_o/\omega$ can be approximated as $\frac{(\Delta\omega)_x}{\omega_o} = \frac{1}{Q_L(x,\beta)}$. Therefore, the magnitude of the input reflection coefficient measured at $-x$-dB return loss is related to the $Q_L(x,\beta)$

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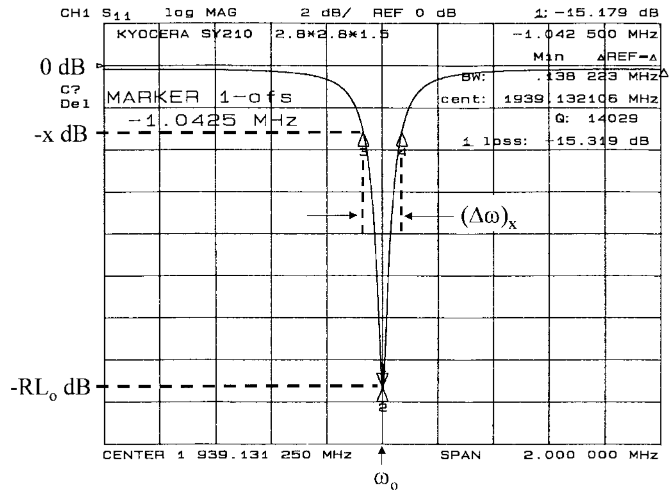

Fig. 2. A typical one-port $|S_{11}|$ measurement of a microwave resonator. Shown here is the measured $|S_{11}|$ for a dielectric resonator showing a Q_u of 32 810.

by

$$|\rho|_x^2 = \frac{[1-\beta]^2 + [Q_u/Q_L(x,\beta)]^2}{[1+\beta]^2 + [Q_u/Q_L(x,\beta)]^2} \tag{2}$$

and $x = -20\log|\rho|_x$. Consequently, the unloaded Q can be expressed simply as

$$Q_u = Q_L(x,\beta)F(x,\beta) \tag{3}$$

where $F(x,\beta)$ is the "mapping function" defined by

$$F(x,\beta) = \sqrt{\frac{(1+\beta)^2|\rho|_x^2 - (1-\beta)^2}{1-|\rho|_x^2}}. \tag{4}$$

Note that both $Q_L(x,\beta)$ and $F(x,\beta)$ are functions of β and x, but Q_u is, in principle, independent of either. At ω_o, where $\Omega = 0$, the return loss is simply

$$\mathrm{RL}_o = -20\log\left|\frac{1-\beta}{1+\beta}\right|. \tag{5}$$

Alternatively, one can write the mapping function $F(x,\beta)$ as

$$F(x,\beta) = \frac{2}{1\mp\rho_o}\sqrt{\frac{|\rho|_x^2-\rho_o^2}{1-|\rho|_x^2}}, \tag{6}$$

where ρ_o is the magnitude of the input reflection coefficient at ω_o given by $\rho_o = 10^{-\mathrm{RL}_o/20}$. The "$-$" and "$+$" signs in (6) correspond to the over-coupled $(\beta > 1)$ and under-coupled $(\beta < 1)$, respectively. These two solutions can be easily distinguished by inspecting the response circle in the Smith Chart from a modern vector network analyzer [3], [4]. Basically, a large response circle enclosing the origin of the Smith Chart signifies an over-coupled case; for under-coupling, the response circle is small and excludes the origin.

With only a scalar network analyzer available, one can still employ this technique by observing the change in RL_o while perturbing the input coupling [11]. For example, an increase in RL_o with increasing coupling signifies an over-coupled response. In the worst-case scenario, where the direction of the coupling strength could not be identified, one would simply take additional data points and calculate the Q_u under both assumptions. Only the correct set of calculation would provide a consistent Q_u value. This method is extremely easy to follow because it does not require any curve fitting or complicated mathematics.

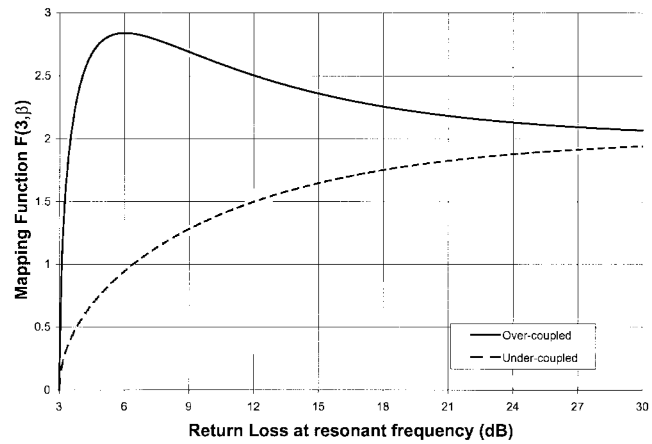

Fig. 3. Mapping function $F(x,\beta)$ as a function of return loss at resonant frequency for $x = 3$ dB.

The mapping function $F(x,\beta)$ is plotted in Fig. 3 for the case of $x = 3$ dB. Clearly, similar curves can be constructed easily using (6) for any value of x. Once the chart or table is developed (or programmed in a pocket calculator), evaluating Q_u is as easy as: 1) read $Q_L(x,\beta)$ directly from network analyzer for any chosen value of x (or calculate $\omega_o/(\Delta\omega)_x$ directly if the network analyzer does not provide an automatic readout); 2) determine whether the resonator is over- or under-coupled by inspecting the Smith Chart (or by observing the changes in return loss while varying the input coupling); and 3) look up the appropriate chart or table for $F(x,\beta)$ at the observed RL_o level [or compute using (6)].

The Q_u is simply the product of $Q_L(x,\beta)$ and $F(x,\beta)$. This procedure is much simpler than other available procedures because no additional mathematical manipulation is required once the function $F(x,\beta)$ is generated as a look-up table or plot. The peak at 6 dB in Fig. 3 is intrinsic to the over-coupled condition of (6). In fact, one can easily show from (6) that there is always a maximum at $\mathrm{RL}_o = 2x$. For example, $F(2,\beta > 1)$ has a peak at $\mathrm{RL}_o = 4$ dB, $F(5,\beta > 1)$ has a peak at $\mathrm{RL}_o = 10$ dB. This suggests that, in practice, one should arrange the input coupling such that $\mathrm{RL}_o > 2x$, especially if a scalar network analyzer is to be used.

IV. External Q Measurement and Tuning

External Q (Q_e) characterizes the coupling between a microwave resonator and the external circuit. In many practical applications, the absolute values of L, C, and r_A in Fig. 1 are not important. In those cases, one can normalize $\omega_o L$ to be one (i.e., $L = 1/\omega_o$ and $C = 1/\omega_o$) and write $Q_e = 1/r_A$. For an all-pole microwave filter, r_A can be expressed in terms of the low-pass prototype parameters as

$$r_A = \frac{w}{g_o g_1} \tag{7}$$

where w is the filter fractional bandwidth. It is also a common practice to use normalized input resistance or conductance in filter design because they are bandwidth independent, yet can be easily scaled with absolute or fractional bandwidth. The normalized input coupling resistance is defined by

$$R_A = \frac{r_A}{w} = \frac{1}{g_o g_1} \tag{8}$$

which is a function of the low-pass prototype parameters only. This normalized resistance has been used in filter design with advanced features for satellite communication applications [17]–[19]. It is

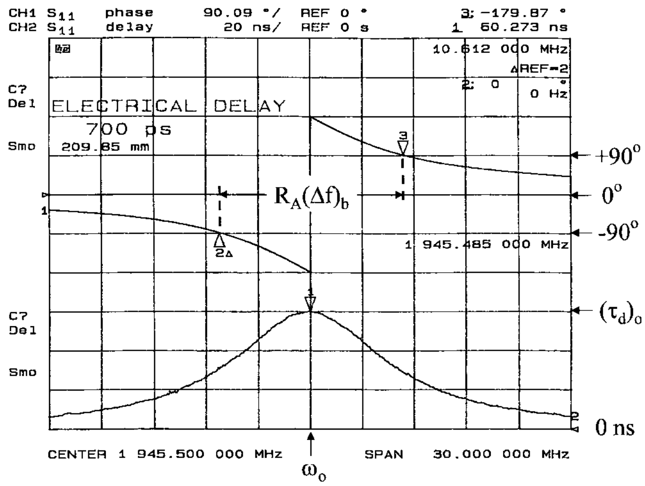

Fig. 4. Measured input coupling in terms of S_{11} phase and time delay for a 0.5% fractional bandwidth dielectrically loaded cavity filter. The electrical delay is adjusted so that (12) is valid.

particularly useful for designing quasi-elliptic function filters or filters with asymmetric response. In those cases, analytical expression for the low-pass prototype does not exist. However, the coupling resistance can still be obtained by a rigorous filter-synthesis procedure [20], [21] or simply by filter-response optimization based on properly constructed equivalent networks.

Again, the equivalent circuit of Fig. 1 is used to measure r_A. Usually, the Q_u is much greater than the Q_e (or equivalently, $r \ll r_A$) for practical applications. Therefore, the reflection coefficient can be simplified to

$$\rho = -\frac{r_A - j\Omega}{r_A + j\Omega}. \tag{9}$$

It is evident from the above equation that at $\Omega = \mp r_A$, the measured phases are $\pm 90°$, respectively. Recall that $\Omega = \frac{\Delta\omega}{\omega_o} = \frac{\Delta f}{f_o}$, the measured bandwidth Δf at $\pm 90°$ can be written as

$$\Delta f_{\pm 90°} = r_A f_o = \frac{f_o}{Q_e} = R_A(\Delta f)_b \tag{10}$$

where $(\Delta f)_b$ is the absolute bandwidth of the filter. For narrow-band filter applications, $R_A(\Delta f)_b$ is the direct measured parameter, as shown in Fig. 4.

With a modern vector network analyzer, one could also measure the input coupling through the time delay of the reflection coefficient. Explicitly, the time delay of the one-port measurement at ω_o can be written as

$$\tau_{d_o} = -\left.\frac{\partial \phi}{\partial \omega}\right|_{\omega_o} = \frac{2}{\pi R_A (\Delta f)_b}. \tag{11}$$

In special cases where Chebyshev prototypes are used and $g_o = 1$, (11) is reduced to the familiar expression of

$$\tau_{d_o} = \frac{2 g_1}{\pi (\Delta f)_b}. \tag{12}$$

Measurement of the time delay might seem redundant because the coupling resistance can be evaluated from (10) with a phase measurement. However, in a practical environment, the reference plane A'-A' of the coupling structure corresponding to the ideal circuit in Fig. 1 is not always precisely identified. To locate the correct reference plane, the delay and phase of the network analyzer should be readjusted until a consistent value of the measured input coupling from (10) and (11) is obtained, as illustrated in Fig. 4.

TABLE I
MEASURED $Q_L(x, \beta)$ AND CALCULATED Q_u OF A DIELECTRIC RESONATOR

f_o (MHz)	RL_o (dB)	x (dB)	$Q_L(x,\beta)$	β (coupling)	Q_u
1939.28	4.26	1	18,499	0.24 (<1, under)	32,765
1939.28	4.26	2	31,612	0.24 (<1, under)	32,658
1939.28	4.26	3	52,201	0.24 (<1, under)	32,571
1939.26	5.05	3	41,400	0.28 (<1, under)	32,653
1939.22	10.20	3	23,766	0.53 (<1, under)	32,746
1939.20	15.60	3	19,615	0.72 (<1, under)	32,751
1939.18	20.00	3	18,170	0.82 (<1, under)	32,782
1939.18	49.20	3	16,415	1.01 (≈1, critical)	32,811
1939.17	20.00	3	14,948	1.22 (>1, over)	32,828
1939.14	15.20	3	14,029	1.42 (>1, over)	32,988
1939.11	10.17	3	12,714	1.90 (>1, over)	33,212
1939.01	5.18	3	11,994	3.45 (>1, over)	33,628
1939.01	4.20	3	13,222	4.22 (>1, over)	33,961
1939.01	4.20	2	7906	4.22 (>1, over)	33,995
1939.01	4.20	1	4509	4.22 (>1, over)	34,042

TABLE II
MEASURED $Q_L(x, \beta)$ AND CALCULATED Q_u OF A COAXIAL RESONATOR

f_o (MHz)	RL_o (dB)	x (dB)	$Q_L(x,\beta)$	β (coupling)	Q_u
2510.10	4.52	2	591	0.25 (<1, under)	643
2510.10	4.52	3	965	0.25 (<1, under)	660
2500.57	13.87	3	269	1.5 (>1, over)	648
2500.57	13.87	5	400	1.5 (>1, over)	636
2500.57	13.87	8	669	1.5 (>1, over)	627
2502.75	1.51	1	83	11 (>1, over)	682

V. MEASUREMENT RESULTS

The accuracy of the procedure in evaluating Q_u is tested with various resonators for Q_u ranging from orders of 10^2 to 10^4. Table I summarizes the result for a high-Q_u dielectric resonator tested under various couplings. The value of β is not required for the Q_u determination, only provided here as a reference. Most data points in Table I agree with the critically coupled Q_u value of 32 810 to within 3%. Errors are expected to be lower (<1.5%) if one follows the recommendation of choosing only $\mathrm{RL}_o > 2x$ for the measurement.

The same procedure has also been applied to a relatively low Q_u miniaturized coaxial resonator. The average Q_u was evaluated to be 643, ±3% as summarized in Table II. Data point with the smallest value of x (1.51 dB) is provided as a reference only. In practice, one should always use larger values of RL_o and x to evaluate the Q_u. Nonetheless, the Q_u estimated from this level of RL_o (1.51 dB) is still quite acceptable (about 6% error).

Fig. 5 shows the measured input coupling for a 0.5% fractional bandwidth quasi-elliptical filter at $f_o = 1946$ MHz. The electrical delay of the network analyzer is adjusted so that the correct reference plane is located and the measured $\Delta f_{\pm 90°}$ and time delay follow (11). Using the described techniques, a six-pole TE01 mode dielectrically loaded cavity filter is built and the final performance is shown in Fig. 5.

VI. SUMMARY

A simple empirical technique for characterizing the unloaded Q and external Q of a high Q_u microwave resonator is presented. The

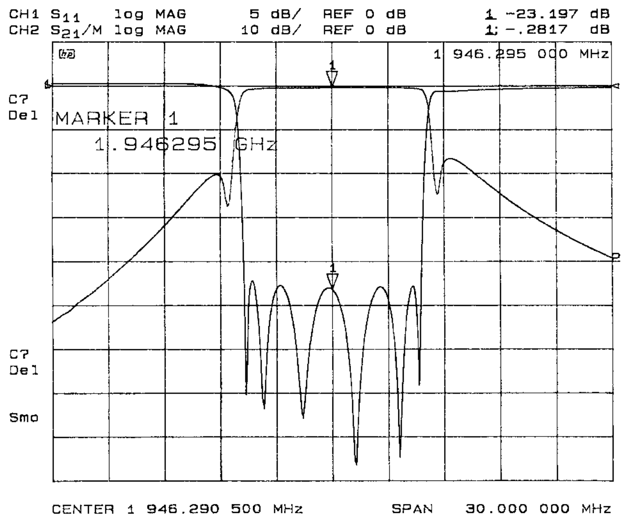

Fig. 5. Final performance of a 0.5% fractional bandwidth dielectric-resonator filter utilizing the described techniques. The realized unloaded Q of the filter is about 25 000.

method we proposed to evaluate the unloaded Q of the microwave resonator is based on a one-port measurement of the generalized loaded Q. By far, this is the simplest method which involves only a look-up table or a chart, as demonstrated. The time delay of the one-port reflection coefficient at the resonant frequency is derived and related to the coupling resistance or the external Q. Combined with the measured phase of the reflection coefficient, one can adjust the reference plane of the network analyzer correctly such that the measured response coincides with the ideal and simulated counterpart. Consequently, the accuracy of the measured coupling is guaranteed. The techniques are proven to be very useful through various experiments.

References

[1] G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks and Coupling Structures*. New York: McGraw-Hill, 1964.

[2] R. E. Collin, *Foundations for Microwave Engineering*. New York: McGraw-Hill, 1966.

[3] C. G. Montgomery, *Technique of Microwave Measurements* (MIT Radiation Laboratory Series 11). New York: McGraw-Hill, 1951, pp. 330–342.

[4] E. L. Ginzton, *Microwave Measurement*. New York: McGraw-Hill, 1957, ch. 9.

[5] ——, "Microwave Q measurements in the presence of coupling losses," *IEEE Microwave Theory Tech.*, vol. MTT-6, pp. 383–389, Oct. 1958.

[6] A. Singh, "An improved method for the determination of Q of cavity resonator," *IEEE Microwave Theory Tech.*, vol. MTT-6, pp. 155–160, Apr. 1958.

[7] W. P. Wheless Jr. and D. Kajfez, "Experimental characterization of multimoded microwave resonators using automated network analyzer," *IEEE Microwave Theory Tech.*, vol. MTT-35, pp. 1263–1270, Dec. 1987.

[8] D. Kajfez, "Graphical analysis of Q circuits," *IEEE Microwave Theory Tech.*, vol. MTT-11, pp. 453–454, Sept. 1963.

[9] D. Kajfez and E. J. Hwan, "Q-factor measurement with network analyzer," *IEEE Microwave Theory Tech.*, vol. MTT-32, pp. 666–670, July 1984.

[10] D. Kajfez, "Linear fractional curve fitting for measurement of high Q factors," *IEEE Microwave Theory Tech.*, vol. 42, pp. 1149–1153, 1994.

[11] ——, "Q factor measurement with a scalar network analyzer," *Proc. Inst. Elect. Eng.*, vol. 142, pp. 369–372, Oct. 1995.

[12] M. C. Sanchez, E. Martin, and J. M. Zamarro, "New vectorial automatic technique for characterization of resonators," *Proc. Inst. Elect. Eng.*, vol. 136, pp. 147–150, Apr. 1989.

[13] ——, "Unified and simplified treatment of techniques for characterizing transmission, reflection and absorption resonators," *Proc. Inst. Elect. Eng.* vol. 137, pp. 209–212, Aug. 1990.

[14] E.-Y. Sun and S.-H. Chao, "Unloaded Q measurement-the critical-points method," *IEEE Microwave Theory Tech.*, vol. 43, pp. 1983–1986, Aug. 1995.

[15] R. Levy, "A generalized design technique for practical distributed reciprocal ladder networks," *IEEE Microwave Theory Tech.*, vol. MTT-21, pp. 519–526, Aug. 1973.

[16] J.-F. Liang, X,-P. Liang, K. A. Zaki, and A. E. Atia, "Dual-mode dielectric or air-filled rectangular waveguide filters," *IEEE Microwave Theory Tech.*, vol. 42, pp. 1330–1336, July 1994.

[17] A. E. Atia and A. E. William, "New types of waveguide bandpass filters for satellite transponders," *COMSAT Tech. Rev.*, vol. 1, no. 1, p. 21, Fall 1971.

[18] ——, "Narrow-bandpass waveguide filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 258–265, Apr. 1972.

[19] S. J. Fiedziuszko, "Dual-mode dielectric resonator loaded cavity filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1311–1316, Sept. 1982.

[20] A. E. Atia, A. E. William, and R. W. Newcomb, "Narrow-band multi-coupled cavity synthesis," *IEEE Trans. Circuits Syst.*, vol. CAS-21, pp. 649–655, Sept. 1974.

[21] H. C. Bell Jr., "Canonical asymmetric coupled-resonator filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1335–1340, Sept. 1982.