Worksheet 6: Applications of matrix determinant

Example 0.42. Find the determinant of

$$\mathbf{A} = \begin{bmatrix} 4 & -7 & -3 \\ 6 & 0 & -5 \\ 2 & 7 & -2 \end{bmatrix}$$

and use it to determine

(a) if the columns of $\mathbf{A}$ are linearly independent;

(b) if the linear transformation $f(\mathbf{x}) = \mathbf{A}\mathbf{x}$ is one-to-one, or onto, or both.

Example 0.43. Use Cramer's rule to solve the following equation

$$\begin{aligned} 5x_1 + 7x_2 &= 3 \\ 2x_1 + 4x_2 &= 1 \end{aligned}$$

Example 0.44. Use the adjoint to find the inverse of the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$$

Example 0.45. Find

(a) the area of the parallelogram spanned by $\mathbf{a}_1 = (1, 2)^T, \mathbf{a}_2 = (3, 4)^T$

(b) the volume of the parallelepiped spanned by $\mathbf{a}_1 = (1, 0, 0)^T$, $\mathbf{a}_2 = (0, 1, 0)$, and $\mathbf{a}_3 = (1, 1, 1)$.