

Worksheet 5: Matrix determinants

Example 0.34. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Find the $(1, 3)$ -submatrix, minor, and cofactor of \mathbf{A} .

Example 0.35. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Compute the determinant of this matrix by using a cofactor expansion along (a) the 2nd row or (b) the 3rd column.

Example 0.36. Compute the determinant of

$$\mathbf{A} = \begin{bmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5 & -8 & 0 & 3 \\ 0 & 5 & 0 & -6 \end{bmatrix}$$

Example 0.37. Find the determinants of

$$\mathbf{A} = \begin{bmatrix} 5 & & & \\ & 2 & & \\ & & 1 & \\ & & & -6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 5 & 0 \\ 7 & 8 & 9 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{bmatrix}$$

Example 0.38. Verify the property $\det(\mathbf{AB}) = \det(\mathbf{A}) \det(\mathbf{B})$ using the following matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix}$$

Example 0.39. Compute

$$\begin{vmatrix} 1 & -3 & 1 & -2 \\ 2 & -5 & -1 & -2 \\ 0 & -4 & 5 & 1 \\ -3 & 10 & -6 & 8 \end{vmatrix}$$

by performing only row replacement operations.

Example 0.40. Compute

$$\begin{vmatrix} 1 & -3 & 1 & -2 \\ 2 & -5 & -1 & -2 \\ 0 & -4 & 5 & 1 \\ -3 & 10 & -6 & 8 \end{vmatrix}$$

by performing only column replacement operations.

Example 0.41. Compute $\begin{vmatrix} 0 & 2 & 6 \\ 1 & -3 & -3 \\ -1 & 0 & 9 \end{vmatrix}$