Worksheet 12: Similar matrices and diagonalization

Example 0.82. Verify that

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\mathbf{B}} = \underbrace{\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}^{-1}}_{\mathbf{P}^{-1}} \underbrace{\begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}}_{\mathbf{P}}$$

This shows that $\mathbf{A}, \mathbf{B}$ are similar to each other.

Example 0.83. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Show that they have the same characteristic polynomial and thus the same eigenvalues, but they are not similar.

Example 0.84. The matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}$$

is diagonalizable because

$$\underbrace{\begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}}_{\mathbf{A}} = \underbrace{\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} 3 & \\ & -1 \end{pmatrix}}_{\mathbf{D}} \underbrace{\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}^{-1}}_{\mathbf{P}^{-1}} \longleftarrow \text{eigendecomposition}$$

but the matrix

$$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$$

is not (we will see why later).

Example 0.85. For the diagonalizable matrix $\mathbf{A}$ in the preceding example, find its 10th power, i.e., $\mathbf{A}^{10}$.

Example 0.86. The matrix $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$ is not diagonalizable because it has one distinct eigenvalue $\lambda_1 = 1$ with $a_1 = 2$ and $g_1 = 1$ (only one linearly independent eigenvector).

Example 0.87. Is the following matrix diagonalizable? If yes, find the eigendecomposition.

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 1 & -1 \\ -2 & 2 & 4 \end{bmatrix}.$$

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Example 0.88. Is the following matrix diagonalizable? If yes, find the eigendecomposition.

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 1 & 1 \\ -2 & 2 & 4 \end{bmatrix}.$$