Perturbations in Lower Uranian Orbit Review

a project presented to
The Faculty of the Department of Aerospace Engineering
San José State University

in partial fulfillment of the requirements for the degree
Master of Science in Aerospace Engineering

by

Zaid Karajeh

December 2017

approved by

Dr. Jeanine Hunter
Faculty Advisor

Perturbations in Lower Uranian orbit Review

Karajeh Z.[1]
San Jose State University, San Jose, California, 95116

Uranus is almost a mystery to many of the scientists and engineers on Earth today. Its existence has been known for centuries, yet the planet has been largely unexplored and thus misunderstood. This paper describes two methods for sending a spacecraft from Earth to Uranus. First, a simple Hohmann transfer from LEO to LUO. Second, a flyby assist at Jupiter via two Hohmann transfers. The results of this paper describe why a flyby assist is the ideal option for a mission to Uranus and how it optimizes the Delta V requirement, in comparison to the more expensive single Hohmann. This paper also describes the tradeoff for conducting a flyby maneuver. The methods used to produce these results are explained in detail. The N-body analysis portion of this investigation also found that propagation did occur on the a spacecraft, the size of Voyager 2. The MATLAB script developed for this analysis has been verified and the results are acceptable.

Nomenclature

a	=	semi-major axis
e	=	eccentricity
GM	=	standard gravitational parameter
m	=	mass
r	=	radius
r_a	=	distance to apoapsis
r_p	=	distance to periapsis
t	=	time
V	=	velocity
V_∞	=	hyperbolic excess velocity
$\forall$	=	volume of a sphere
β	=	asymptote angle
Δ	=	aim radius
θ	=	phase angle
ρ	=	density
τ	=	period of orbit
ω	=	angular velocity

I. Introduction

Uranus is often over looked as a planet of interest. Its distance from the Earth has played a pivotal role in the prevention of further research into the Ice Giant. Although no one planet in our Solar System should be considered more important than the others, it is often seen that planets like Mars and Saturn are higher points of focus. Uranus is unique in that its irregular orbit, polar locations, and rings could provide a better insight on the history of our solar system. Further investigation into the planet will help provide a basis for possible future missions. Investigation into the nature of Uranus and the nature of natural satellites currently orbiting the Ice Giant will help determine if Oranocentric flight will be possible.

II. Objective

The general objective of this paper is to provide a basis for future missions to Uranus. This research will be observed in two major parts. Firstly, a discussion will be presented on the topology and history of Uranus, its rings, moons, and atmosphere. A description will be presented on the determination of a launch window, trajectory, and delta V budget necessary to propel a satellite from Earth Parking Orbit

[1] Masters Student, Department of Aerospace Engineering, 1 Washington Square, San Jose, CA,95192, Student.

to Lower Uranian Orbit (LUO). Secondly, this research will investigate the gravitational perturbations produced on the space vehicle due to the rings of the Ice Giant; this will be described through the development of an N-Body model.

III. Background

A. History

Sir William Herschel first discovered Uranus in the late 18th Century with a home-made telescope. At the time of its discovery, Uranus was deliberated as different celestial bodies ranging from a star to a comet. When Herschel had confirmed the body as a plant of our solar system, he and other astronomers failed to recognize its rings. In 1977, James Elliot, Edward Dunham, and Jessica Mink initially found a total of nine rings in orbit around Uranus via occultation. In 1986 when Voyager flew by Uranus, it had discovered two additional rings. Finally, in 2005, Hubble detected the last two rings of the Ice Giant totaling thirteen. There have been virtually no missions dictated toward the research of Uranus. As it stands, we have never sent any satellites to Uranus, with the exception of the Voyager 2 flyby. This paper and the research provided will potentially outline the necessary information needed to sustain a satellites orbit to LUO.

B. General Topology

The topology of Uranus is unique in that it is the only planet in the solar system that exhibits a tilted axis of rotation. Uranus faces a 97.7-degree axial tilt with retrograde rotation. Most planets in the solar system exhibit an almost 0 degree axial tilt; this becomes interesting for the development and design of a mission to the Ice Giant. If a satellite were to travel along plane of the solar system, or ecliptic plane, into oranocentric orbit it would face a polar orbit; it important to note that Uranus is 0.773 degrees inclined to the ecliptic. Unlike other planets in the solar system, Uranus' poles can face extreme exposure to the sun due to the abnormal orientation of the planet. For the purposes of this paper, the mission will intend for LUO flight in polar orbit as to stay aligned with the ecliptic plane.

Uranus also experiences an eighty-four year heliocentric orbit; this means it also faces seasonal changes once every estimated twenty-one years. When Voyager 2 passed the south pole of Uranus in 1986, scientists failed to recognize at the time that the reason for its lackluster appearance was due to lack of solar exposure. The Southern Hemisphere, at the time, was in the midst of winter. As will be discussed in more depth in the coming sections, Uranus has thirteen rings and twenty-seven moons with some moons embedded within particular rings. The estimated Perihelion of Uranus is 2,735,569,00 km whereas its Aphelion is 3,006,390,000 km. For the purposes of sustaining lower orbit flight, the atmosphere was also investigated; the constituents of the atmosphere are as follows, 82.5% hydrogen (H_2), 15.2% Helium (He), and 2.3% Methane (CH_4) as well as various trace compounds. The table attached provides additional information on the general topology of Uranus.

Table 1. General Orbit Information on Uranus.

Semimajor axis	25,559E6 km
Semiminor axis	24,973E6 km
Volume	5.914E13 km^3
Average density	1270 kg/m^3
Gravitational Acceleration	8.69 m/s^2
Rings	13 rings
Moons	27 observed
Sol	17 Earth hours 42 minutes
Angular velocity at equator	9315 km/hr
Rotation	Retrograde
Sidereal Period	83.74 Earth years

Orbital Velocity	6.83 km/s
Perihelion	2,735,569,00 km
Aphelion	3,006,390,000 km
Orbital eccentricity	0.04717
Orbital inclination to the eliptic	0.77 degrees
Obliquity (inclination of equator to Orbit)	97.77 degrees

C. Rings

Uranus' rings contain a history of their own. Because of the nature of their orbit around the planet, scientists believe that the rings of Uranus formed much later. There are currently thirteen known rings of Uranus, Zeta, 6,5,4, Alpha, Beta, Eta, Gamma, Delta, Lambda, Epsilon, Mu, and Nu. The table below provides information on the distances between the rings and the surface of Uranus.

Table 2. Distances and Eccentricities of the Rings of Uranus.

Rings	Distance from surface to inner edge	Approximate width	Eccentricity
1986U2R (also called Zeta)	38,000 km	2,500 km	-
6	41,837 km	1.5 km	0.001
5	42,235 km	2 km	0.0019
4	42,571 km	2.5 km	0.001
Alpha	44,718 km	4-10 km	0.0008
Beta	45,661 km	5-11 km	0.0004
Eta	47,179 km	1.6 km	-
Gamma	47,626 km	1-4 km	0.0001
Delta	48,303 km	3-7 km	-
Lambda	50,024 km	2 km	-
Epsilon	51,149 km	20-96 km	0.0079

Rings	Distance from surface to inner edge	Approximate width	Eccentricity
R/2003 U 2 (Mu)	67,300 km	3,800 km	0
R/2003 U 1 (Nu)	97,700 km	17,000 km	0

There does seem to be a difficulty in understanding what the rings are actually composed of. What is known is that the inner eleven rings of Uranus, the dark rings, are composed of dark ice. The outer two rings, the bright red rings, are composed of dust on the scale of micrometers. The objects populating the inner rings are on the magnitude of one to ten meters in size. There is much deliberation and speculation as to what the rings are composed of in terms of object size, density, and material composition. The rings of the planet are nearly circular in shape, as displayed by their eccentricities.

D. Moons

There are currently twenty-seven known moons of Uranus. Each moon is named after a character created by William Shakespeare or Edgar Allen Poe. The moons, with their distances from the center of

Uranus, are listed in table 3 from closest to farthest. The moons Portia and Mab are embedded in the rings Nu and Mu respectively. The other moons orbit Uranus between the rings, with the exception of Miranda which orbits outside the farthest ring, Mab. With that said, the moons of Uranus also exhibit inclination angles to the Uranian plane.

Table 3. Moons and their distances from the center of Uranus.

Moons	Distance (Km)
Cordelia	49,752
Ophelia	53,764
Bianca	59,165
Cressida	61,767
Desdemona	62,659
Juliet	64,630
Portia	66,097
Rosalind	69,927
Cupid	74,800
Belinda	75,255
Perdita	76,420
Puck	86,004
Mab	97,734
Miranda	129,390
Ariel	191,020
Umbriel	266,300
Titania	435,910
Oberon	583,520
Francisco	4,276,000
Caliban	7,231,000
Stephano	8,004,000
Trinculo	8,504,000
Sycorax	12,179,000
Margaret	14,345,000
Prospero	16,256,000
Setebos	17,418,000
Ferdinand	20,901,000

E. Notes

It is important to note that Uranus is still a mystery to many researchers and engineers in the planetary sciences. Missing information, such as the actual composition of the rings of Uranus and definitive size of the objects that populate those rings will all be subject to further educated assumptions. Uranus has been largely unexplored and it is the hope of this paper to develop a feasible mission to understand whether it would be possible to send a spacecraft to the Ice Giant.

IV. Hohmann Transfer

In order to achieve the ultimate goal of this paper, an N-body problem relating the orbit of the space vehicle to the objects found in Uranus' rings, the space vehicle must first arrive at its destination. Firstly, It is decided that a Hohmann transfer would be conducted, as the mission is not time sensitive. In order to conduct the Hohmann transfer, certain parameters have been found and certain assumptions made to simplify the problem in this section. The transfer ellipse and hyperbola parameters have been defined and will be shown in later sections. For the sake of this analysis, we have assumed a coplanar transfer with both the Earth and Uranus level to the ecliptic plane. This paper will outline a simple Hohmann case before further outlining to a more complicated orbital assist to Uranus.

A. Transfer Ellipse

In order to define the transfer ellipse, a number of parameters must be found. Firstly, the perihelion and apohelion of the ellipse, which are the distance from the sun to the earth and the sun to Uranus respectively, must be defined. Citing NASA's planetary fact sheet, it is found that the values are as follows:

$$r_p = Perihelion\ of\ transfer\ ellipse\ (Earth) = 147.09 \times 10^6\ Km$$
$$r_a = Apohelion\ of\ transfer\ ellipse\ (Uranus) = 2{,}741.3 \times 10^6\ Km$$

The semi major axis of the transfer ellipse is defined by equation 4.1 which is, by definition, the average of the perihelion and apohelion.

$$a_{Transfer} = \frac{1}{2}\left(r_p + r_a\right) \quad (4.1)$$

The shape of the ellipse is defined by it's eccentricity which is shown in equation 4.2.

$$e_{Transfer} = \frac{r_a - r_p}{r_a + r_p} \quad (4.2)$$

The values yielded by both the semi major axis and the eccentricity, for the case of this Hohmann transfer, are 1444195000 Km and .8981496229 respectively.

B. Definition of the Transfer Hyperbola Upon Departure and Arrival

In order to embark on the transfer ellipse, the transfer hyperbola must be defined. This section will define the hyperbolic excess velocity necessary to rendezvous with Uranus and depart from Earth, define both the asymptote angles for Earth and Uranus, define the aiming radii, and the delta V for both hyperbolas.

It is firstly important to define the hyperbolic excess at both planets. The hyperbolic excess velocity is defined in equations 4.3 and 4.4.

$$V_{\infty,Uranus} = \sqrt{\frac{GM_{sun}}{r_a}}\left[1 - \sqrt{\frac{2r_p}{r_p + r_a}}\right] \quad (4.3)$$

$$V_{\infty,Earth} = \sqrt{\frac{GM_{sun}}{r_p}}\left[\sqrt{\frac{2r_a}{r_a + r_p}} - 1\right] \quad (4.4)$$

Substituting the values we know for r_p, r_a, and GM_{sun}, we obtain a hyperbolic excess of 4.658432669 Km/second at Uranus' sphere of influence and 11.28131883 Km/second at Earth's sphere of influence.

Defining the aiming radii and asymptote angles, as equation 4.5 and 4.6 respectfully show, will allow the spacecraft to neither over nor undershoot the ellipse in transfer.

$$\Delta = r\sqrt{1 + \left(\frac{2GM}{rV_\infty^2}\right)} \quad (4.5)$$

$$\beta = \cos^{-1}\left(\frac{1}{1 + \frac{rV_\infty{}^2}{GM}}\right) \quad (4.6)$$

Manipulating these equations to both Uranus and Earth, we find the following:

$$\Delta_{Earth} = 9500.008853\ Km$$
$$\Delta_{Uranus} = 133898.8019\ Km$$
$$\beta_{Earth} = 71.75360899^\circ$$
$$\beta_{Uranus} = 26.62898572^\circ$$

Lastly, it is imperative to find the ΔV required to transfer from the hyperbola to the ellipse as shown in 4.7 and 4.8.

$$\Delta V_{Uranus} = \sqrt{{V_\infty}^2 + \frac{2GM_{uramus}}{r_p}} - \sqrt{\frac{2GM_{Uranus}}{r_p} - \frac{GM_{Uranus}}{a_e}} \quad (4.7)$$

$$\Delta V_{Earth} = \sqrt{{V_\infty}^2 + \frac{2GM_{Earth}}{r_a}} - \sqrt{\frac{2GM_{Earth}}{r_a} - \frac{GM_{Earth}}{a_e}} \quad (4.8)$$

The values produced from the equations are 6.160793744 Km/s and 7.981264515 Km/s respectful to Uranus and Earth.

C. Ephemeris and Time to Travel

In order to actually launch to Uranus, the ephemeris of both the Earth and Uranus must both be determined with respect to dates of the Julian calendar. It is necessary to determine the time of travel along the transfer ellipse which is determined by equation 4.9.

$$t_{Transfer} = \pi\sqrt{\frac{a^3}{GM_{sun}}} \quad (4.9)$$

This simple calculation yielded an approximate travel time for 15 years. Upon further analyzing that time, it is important to note that Hohmann transfers are low energy and therefore often produce long wait times and relatively slow velocities.

Because the launch of the space vehicle from Lower Earth Orbit is dependent on the ephemeris of both Earth and Uranus, it is necessary to select an "initial day" which will help determine the wait time to launch. The paper has arbitrarily selected the 15th of December 2017 as it's initial day. Referencing NASA JPL's Horizons software, which determines the ephemerides of planets in our solar system, it was found that the ephemeris of Earth and Uranus are as follows:

$$\theta_{Earth,dec\ 15\ 2017} = 2.134649677784732\times10^2 \text{ degrees}$$
$$\theta_{Uranus\ dec\ 15\ 2017} = 3.415039723125511\times10^2 \text{ degrees}$$

In order to calculate the wait time, as described in equation 4.10, both the true anomalies and angular velocities of the planets must be known.

$$t_{wait} = \frac{\Delta\theta}{\Delta\omega} \quad (4.10)$$

A simplified version of the angular velocity equation is presented.

$$\omega = \sqrt{\frac{GM_{sun}}{r^3}} \quad (4.11)$$

Where ω is the angular velocity of the given planet and r is the either the perihelion or apohelion, assuming circular orbit. Solving for the wait time, it is found that the space vehicle must wait 128 Days 5 Hours 59 Minutes 57.02 Seconds to departure. This would mean that the day of launch is on the 22nd of April 2018.

D. Discussion

This analysis provides two notable pieces of information. First, because the Earth travels so much faster relative to Uranus, a launch window for a simple Hohmann transfer will open up frequently. A launch window will open at least once a year to Uranus, granted no flyby maneuvers are conducted. It is also important to note the Delta V required. Although Hohmann transfers are low energy transfers, it is still possible to optimize the Delta V required. Looking at the total Delta V needed to preform this single Hohmann, which is the sum of the Delta V's required, a real mission would try other means to reduce this value. Since these are propulsive Delta V's, parameters such as fuel consumption and cost become problematic. The next section will discuss optimizing the Hohmann transfer by using a flyby assist at Jupiter. The expectation is that Delta V will be reduced but wait time to transfer, and transfer time, will increase significantly.

V. Flyby Assist Through Two Hohmann Transfers

A. Theory Behind the Assist

For the sake of this analysis, a single Hohmann transfer to Uranus might not be enough since it is possible to optimize the Delta V required. It can be thought that simple Hohmann transfers actually provide relatively high delta V's compared to other maneuvers. So, how could this be corrected? There are different maneuvers that can be conducted, and different methods to conducting such maneuvers, but the simplest of these complex transfers would be to preform two Hohmann transfers. The maneuver would require one Hohmann transfer from Earth to a flyby planet and then another from the flyby planet to the target. The flyby assist is essentially a slingshot method, which would effectively produce a lower more optimal, delta V but a longer time of flight.

When selecting the flyby planet, the most commonsensical would be to select Jupiter. Jupiter's shear size and mass would, when the flyby maneuver is conducted, would effectively produce the highest delta V compared to any other planetary option. In comparison to Mars or Saturn, it would simply make the most sense to select the celestial body that is closest and largest to obtain the assist. The analysis shown in this section will discuss three scenarios, with the results displayed below. The scenarios will be as follows, if a satellite were in Lower Earth Orbit on the 12th of May, 2017, how long would it have to wait until it could conduct it's first maneuver to Jupiter in order to flyby to Uranus? If a satellite were in LEO on the 15th of December 2060 how long would a satellite have to wait until it could conduct its first maneuver to Jupiter and then to Uranus? And finally, the wait time required if the satellite were in LEO on the 2nd of April 2180. Next, we will produce the total delta V required to launch to Uranus from Earth via a flyby assist at Jupiter.

B. Assumptions

The assumptions involved in this analysis were conducted to simplify calculations and the understanding of the problem. It is important to note that the Earth, and the other planets included in the analysis, does not naturally follow circular orbits. J2 oblateness is not in effect for any of the planets in this paper. The analyses are also conducted using patched conics, which is purely two-body based. This code also goes to estimate an altitude in which the spacecraft will depart the Earth, flyby at Jupiter, and arrive at Uranus; those altitudes are 500, 200,000 and 60,000 km respectively. 500 km was selected as an ideal departure altitude because that places Earth in LEO. 200,000 km was selected as the flyby altitude because this would be a distance close enough for the spacecraft to be launched from Jupiter to Uranus without being captured by Jupiter. 60,000 km was selected as the arrival altitude for Uranus because it would place the vehicle outside the rings; this would be the starting point for the N-body analysis that will be conducted in future research. Because this is simply two Hohmann transfers, it can also be assumed that the time to transfer from Earth to Uranus via flyby assist is simply the sum of the transfer time from Earth to Jupiter and the transfer time from Jupiter to Uranus. Making a summation from equation 4.9, the total time for transfer is:

$$t_{Total\ transfer} = \sum_{i=1}^{2} \pi \sqrt{\frac{a_i^3}{GM_{sun}}} \tag{5.1}$$

Where the value for total transfer time is 23.412 years.

C. Implementation of MATLAB Script

The MATLAB scripts produced, which can be referenced in Appendix A, B and C of this paper, can be thought of being split into two parts. Part I provides the wait time needed to conduct a flyby maneuver. Part II is a calculator to define the parameters needed to produce the more optimal delta V required. The aforementioned equations in section IV of this paper layout the basics involved in conducting a Hohmann transfer and are implemented in the mentioned code. Remember, the flyby assist produced here is through two simple Hohmann transfers. The most difficult part of conducting these maneuvers is to provide a basis from when a transfer can be conducted from Earth to flyby from Jupiter to the target, Uranus.

D. Explanation of MATLAB

This paper shall begin by describing the code in section A of appendix A. The code begins by describing the parameters needed to produce the necessary results. Firstly the constants will be explained.

As commented, it shows the perihelion and apohelion from Earth to Jupiter. The code also describes the corresponding altitudes the satellite will depart at and arrive at in both Earth and Jupiter orbit. The code also defines the standard gravitational parameters for the Sun, Earth, and Jupiter.

Now the calculations are initialized. The periods of Jupiter, Earth, and the transfer orbit are defined as follows:

$$\tau = \sqrt{\frac{4\pi^2 a^3}{GM_{sun}}} \tag{5.2}$$

Because of our assumption that the Earth and Jupiter are in circular heliocentric orbit, the semi-major axis, a, is equal to the distance of the planets to the sun. This is necessary for calculating the angular velocities of both the Earth and Jupiter. Next, the transfer ellipse from Earth to Jupiter is defined. Using equations 4.1 and 4.2, the semi-major axis and eccentricity are respectively calculated. The calculator then defines the phasing parameters for Earth to Jupiter transit. Firstly, the period of the transfer orbit is calculated using equation 5.1. This is necessary to calculate the θ_{Δ}, the angular distance covered by Jupiter in the time it takes for the spacecraft to travel to the rendezvous location 180 degrees away. The phase angle that Jupiter must be at when the transfer is initiated is then calculated.

It is important to note, that Appendix A section A and Appendix A section B complete the same task for two separate transfer orbits. The former describes transfer to Jupiter from Earth whereas the latter describes transfer to Uranus from Jupiter. The difference between the two codes is that Appendix A section A also discusses the launch window opening by utilizing both scripts.

The next section of this explanation will discuss the methodology involved in determining the launch dates available for the flyby. When producing mission plans for a flyby maneuver, all planets must be some angular distance away from one another so that when departure is initiated, the spacecraft in question does not miss its targets. How is it possible to determine the angular distances of the Earth-Jupiter-Uranus system? There are two methods, one is to look into published data, such as JPL Horizons, that explicitly estimates the exact true anomalies of the planets in question for a large set of dates and attempt to filter out the incorrect true anomalies. The other method, the one implemented in this paper, is to select an arbitrary date and determine the wait time necessary to launch when the planets have arrived at a phase angle that a spacecraft can launch at in which it will not miss its target. The initial true anomalies are also selected from JPL's Horizon's tool.

This is a similar analysis conducted in the simple Hohmann transfer section. Refer to equation 4.10. Wait time is simply the change in phase angle divided by the change in angular velocities.

$$t_{wait} = \frac{\Delta\theta}{\Delta\omega} \tag{4.10}$$

The difference in this analysis is that the calculations are done iteratively. The analysis takes the true anomalies of three dates that this paper observes and continually increases them based off of the angular velocity and time step chosen. Each loop will check the current phase angle between the planets with two to check; one between Earth and Jupiter and one between Jupiter and Uranus. In order to exit the loop and produce the time to wait, both angles must match the desired phases.

In other words, the script looks at the true anomalies of the planets on the given date, and iterates them over the given time step. If the iterated true anomaly extends past a value above 2π radians, it will reset to 0 to keep the value of the iterated true anomaly between 0 and 2π. The code goes into producing the phase angles necessary for phasing between Earth to Jupiter and Jupiter to Uranus. It applies a condition where, if the difference in true anomalies are negative for both sets of planets, it will preform an additional calculation to produce positive values. Finally, at time t when the planets are at their respective true anomalies, $t \times time\ step$ will produce a value when the flyby transfer will be possible.

The code presented in Appendix A section C provides the delta V optimization. The word optimization will be associated with the fact that the delta V produced from the flyby assist will be lower than that of the simple Hohmann transfer.

The altitudes are firstly initialized. As mentioned before, the altitude of Earth has been set to 500km, Jupiter to 200,000 km, and Uranus to 60,000km. Next, the velocities of each planet are calculated in circular heliocentric orbit using the circular velocity equation:

$$V = \sqrt{\frac{GM_{sun}}{r}} \tag{5.3}$$

The code then calculates the transfer ellipse velocities from Earth to Jupiter then from Jupiter to Uranus using the energy equation.

$$V = \sqrt{\frac{2GM}{r} - \frac{GM}{a}} \tag{5.4}$$

Then, the hyperbolic excess velocities and eccentrics are formulated which are inputted into the turning angle calculations. The hyperbolic excess velocities are implemented as:

$$V_\infty = V^{S-Q} - V^{S-P} \tag{5.5}$$

Where V^{S-Q} is the spacecraft velocity with respect to the sun and V^{S-P} is the planet velocity with respect to the sun. From this, hyperbolic eccentricity can be found using:

$$e = 1 + \frac{r_p V_\infty}{GM} \tag{5.6}$$

From this, the turning angle can be calculated using the arcsine of the inverse hyperbolic eccentricity. This equation is particularly important because it is dictated by the radius at periapsis. If the radius is too high or too low, the delta V could be too high.

Finally, the delta V is calculated by using the decomposed parts of the hyperbolic velocity and from that, the three burns at each planet are combined to produce the delta V for the flyby assist.
The above equations are all expressed as scalar quantities, but in the actual implementation of these missions, everything is expressed as vector quantities. The transfer velocity, as expressed in vector notation is as follows. The mentioned equations and the one below are identical, but equation 5.7 is expressed in a sun vector frame.

$$V_{Tr} = \frac{GM}{h} esin\theta\, \widehat{Sy} + \frac{GM}{h} ecos\theta\, \widehat{Sx} \tag{5.7}$$

Where the sign of both components will vary with the positions of the planets for each case. This equation is applicable to both transfer to Jupiter from Earth, and to Uranus from Jupiter. Likewise, the hyperbolic excess velocities can also be expressed as vector quantities. Where the departure from Earth is expressed as:

$$V_{\infty,d,earth} = \sqrt{\frac{2GM_{earth}}{r} - \frac{GM_{earth}}{a}}\, \widehat{Sy} - \sqrt{\frac{GM_{earth}}{r}}\, \widehat{Sy} \tag{5.8}$$

The arrival to Jupiter is expressed as:

$$V_{\infty,a,jupiter} = -\sqrt{\frac{2GM_{earth}}{r} - \frac{GM_{earth}}{a}}\, \widehat{Sy} + \sqrt{\frac{GM_{earth}}{r}}\, \widehat{Sy} \tag{5.9}$$

The departure from Jupiter is expressed as:

$$V_{\infty,d,jupiter} = \sqrt{\frac{2GM_{earth}}{r} - \frac{GM_{earth}}{a}}\, \widehat{Sy} - \sqrt{\frac{GM_{earth}}{r}}\, \widehat{Sy} \tag{5.10}$$

And the Arrival to Uranus is expressed as:

$$V_{\infty,a,uranus} = -\sqrt{\frac{2GM_{earth}}{r} - \frac{GM_{earth}}{a}}\, \widehat{Sy} + \sqrt{\frac{GM_{earth}}{r}}\, \widehat{Sy} \tag{5.11}$$

E. Results of MATLAB CODE

1. Case I, May 12th 2017

The wait time produced for the 12th of May 2017, at midnight, was 43.19 years. This means that approximately 49 years from this initial day is when the launch window will open.

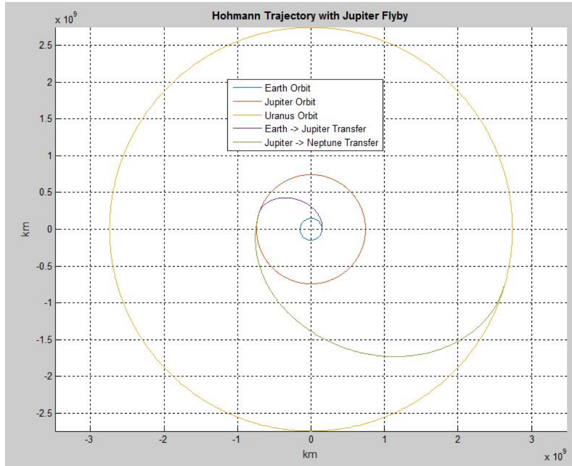

Figure 1. Orbital Assist Around Jupiter to Uranus for the Case of May 12th 2017

The figure above illustrates the orbital assist from Jupiter to Uranus after waiting 43.19 years to conduct the maneuvers. The figure shows transit from Earth to Jupiter and then Jupiter to Uranus. It must be noted that the subsequent figures also display similar maneuvers but at different true anomalies.

2. Case II, December 15th 2060

The wait time produced for the 15th of December 2060, at midnight, was 10.54 years. This means that approximately 10 and a half years from the initial day is when the launch window will open.

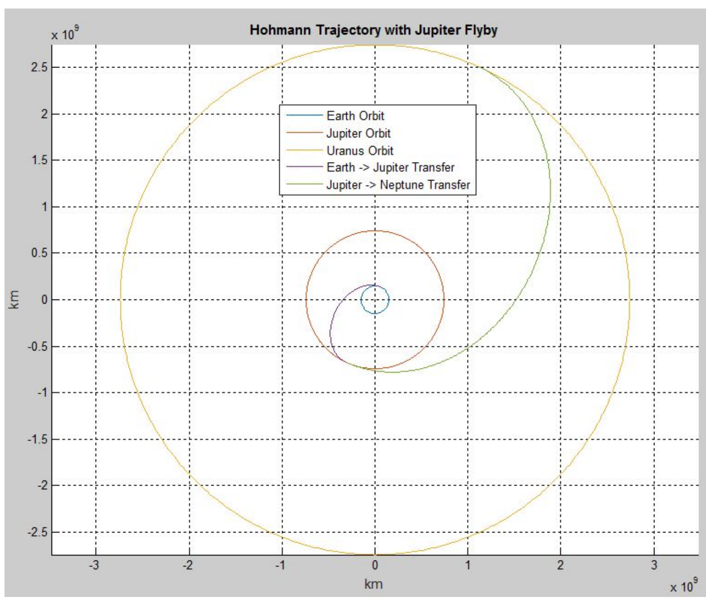

Figure 2. Orbital Assist Around Jupiter to Uranus for the Case of December 15th 2060

3. Case III, April 2nd 2180

The wait time produced for the 2nd of April 2180, at midnight, was 76.85 years. This means that approximately 77 years from the initial day is when the launch window will open.

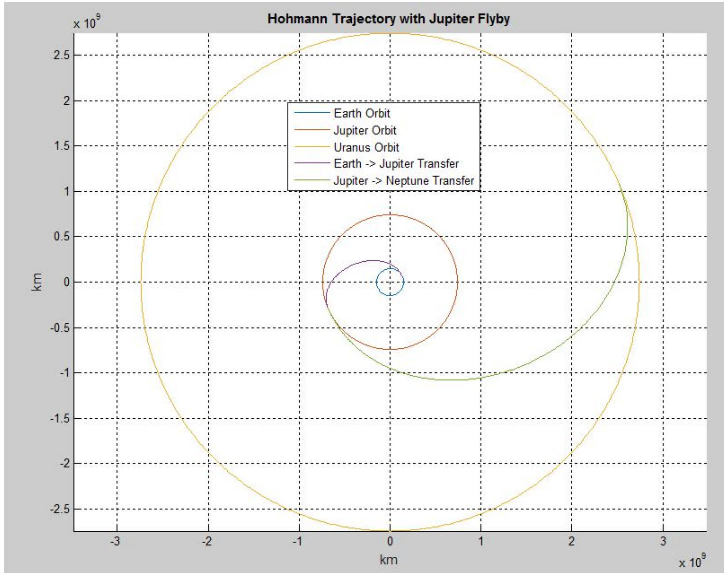

Figure 3. Orbital Assist Around Jupiter to Uranus for the Case of April 2nd 2180

4. Delta V

The results for the Delta V calculator proved to be successful. Setting the altitude of arrival Uranus to 60000 km and the flyby altitude at Jupiter to 200000, the total Delta V results to 13.778 km. The Delta V upon departure of earth is 6.2684 km/s. The Delta V corrected at Jupiter is 3.8531 km/s; this correctional value is the delta V to correct the hyperbola for arrival at Uranus. Finally, the Delta V at Uranus is -3.6564 km/s where the spacecraft must slow down to arrive.

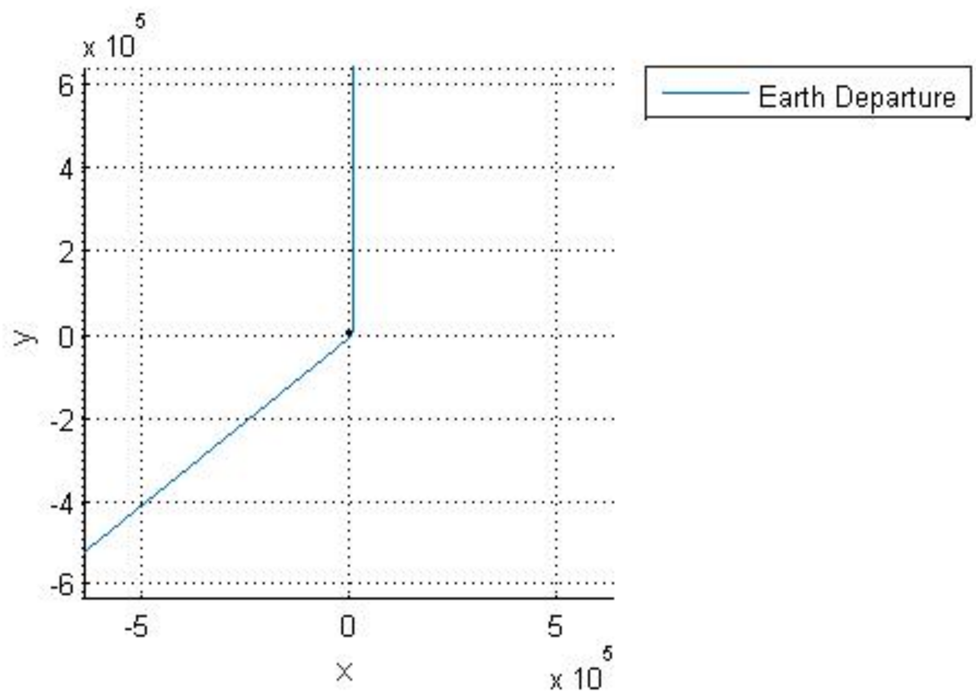

Figure 4. Vector diagram showing departure from Earth

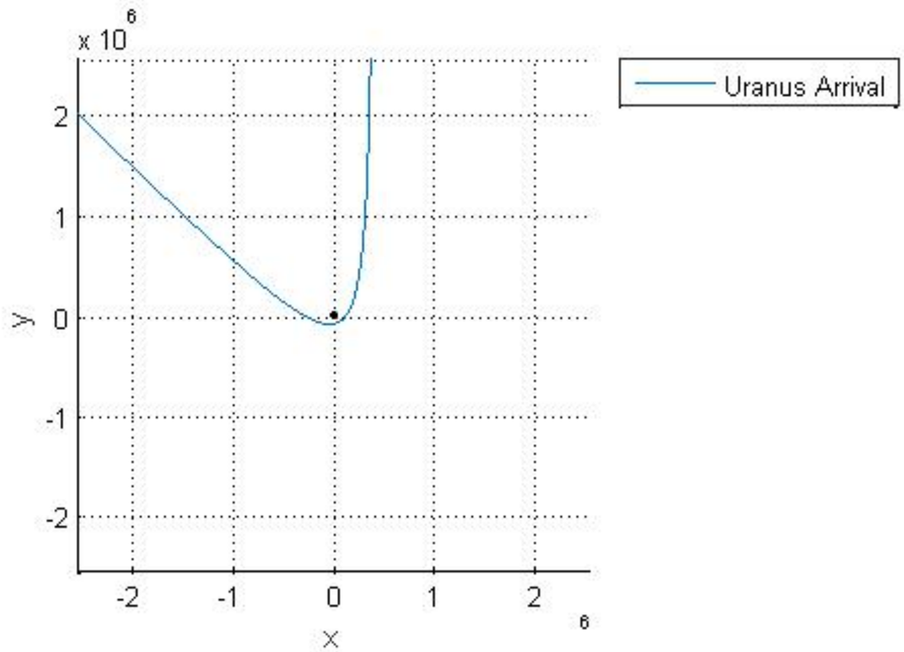

Figure 5. Vector Diagram showing arrival to Uranus

F. Discussion

Firstly, it is important to ask how real missions to the outer planets are conducted and what the results presented from this paper's analysis mean. Although Hohmann transfers are important, because they are low energy transfers and provide low delta V requirements, the wait time needed to conduct the maneuvers and the transfer time makes for a mediocre to poor option. Although the transfer time for the flyby has increased relative to the simple transfer, delta V required has been decreased. Other solutions may be more useful, and it would be of use to investigate other methods of transfer such as Lambert's problem.

VI. N-Body Analysis

A. Explanation of N-Body Problem

When the spacecraft dives through the rings of Uranus into lower ouranocentric orbit, a point of concern is how the objects in these rings will perturb the spacecraft in flight. This becomes an N-Body problem. Two models will be explored when conducting the N-body analysis. The first, and possibly least feasible of the models presented in this text, is to assume the rings as solid objects and assign them effective masses based on random population of objects. The more feasible model is to preform an N-body analysis on a random number of objects in the Zeta ring with respect to the spacecraft and Uranus while in lower ouranocentric orbit.

B. Characterization of Rings

1. Estimation of Mass based on size and density

The rings of Uranus can be characterized by the main constituency of their masses, "blackened ice". This blackened ice is simply methane ice blackened by high-energy particles bombarding off the masses from the magnetosphere. We know that the size of the masses can vary from micrometers to sizes of five to ten meters in diameter. In order to estimate the mass of these objects, we will make two assumptions. Firstly, these masses will be perfectly spherical. Asteroids and other celestial bodies, such as the blackened ice, are not perfect nor are they devoid of cracks and abrasions. To simplify this analysis they will be assumed to be perfectly solid and round. Secondly, these objects will range from five to ten meters in diameter. It is my hope that a MATLAB script will be generated, in the continuation of this paper, that can produce a random number of objects between a range of 500,000 and 1,000,000. The investigation will look into how masses of the specified range in orbit around Uranus will perturb the spacecraft. These objects will be populated with some mass based off of a randomly assigned diameter with respect to the density of methane ice, also known as methane clathrate. This will be done by using the following:

$$\rho = \frac{m}{\forall} \tag{6.1}$$
$$\forall = \frac{4}{3}\pi r^3 \tag{6.2}$$

Where ρ is the density, which is approximately $900\,\frac{kg}{m^3}$ and V is the volume defined for a sphere. With this a simple model for the masses can be expressed:

$$m = \frac{4}{3}\rho\pi r^3 \tag{6.3}$$

C. Velocity of the Rings

As described by Randii Wessen, Jeffrey Cuzzi, and Ellis Miner in "Planetary ring systems", each ring is observed at an effective velocity. Those velocities are presented in table 4 below.

Ring	Velocity (km/s)
Zeta	12.11
6	11.78
5	11.724
4	11.667
Alpha	11.393
Beta	11.274
Eta	11.091
Gamma	11.307
Delta	10.961
Lambda	10.77
Epsilon	10.65
R/2003 U2 (Mu)	9.28
R/2003 U1 (Nu)	7.7

Table. 4 Effective Velocities of Uranus' rings in the planetary reference frame of Uranus.

The masses within the model that will be generated will be assigned the effective velocity of the ring that they will be present in.

D. Ideal Model

The particles within the closest ring, Zeta, will be modeled in a range from one to ten meters in size, as mentioned in the Rings section of this paper. Using methane clathrate's density and the range of sizes mentioned, the following potential candidates for ring particles are as follows:

$m_1 = \frac{4}{3}\rho\pi r_1^3 = 3769.91\ kg$

$m_2 = \frac{4}{3}\rho\pi r_2^3 = 30159.29\ kg$

$m_3 = \frac{4}{3}\rho\pi r_3^3 = 101787.60\ kg$

$m_4 = \frac{4}{3}\rho\pi r_4^3 = 241274.32\ kg$
$m_5 = \frac{4}{3}\rho\pi r_5^3 = 471238.90\ kg$
$m_6 = \frac{4}{3}\rho\pi r_6^3 = 814300.82\ kg$
$m_7 = \frac{4}{3}\rho\pi r_7^3 = 1.29 \times 10^6\ kg$
$m_8 = \frac{4}{3}\rho\pi r_8^3 = 1.93 \times 10^6\ kg$
$m_9 = \frac{4}{3}\rho\pi r_9^3 = 2.75 \times 10^6\ kg$
$m_{10} = \frac{4}{3}\rho\pi r_{10}^3 = 3.80 \times 10^6\ kg$

Based on the current science and knowledge conducted on Uranus' rings, these mass and size values are estimations at best. It will be reiterated, it is assumed that all particles in the rings are composed of methane clathrate and that the particles within Zeta all range from one to ten meters in diameter. Based on this information, a model of Uranus and its Zeta ring have also been modeled in MATLAB. The widths of each ring have been listed in table 2. Zeta's width is approximately 2500 km. Each particle will be randomly positioned somewhere on the ring and given the effective ring velocity. Their masses will be randomized as well based on size, as previously expressed. Figures 4 and 5 show the geometry of Uranus and the Zeta ring.

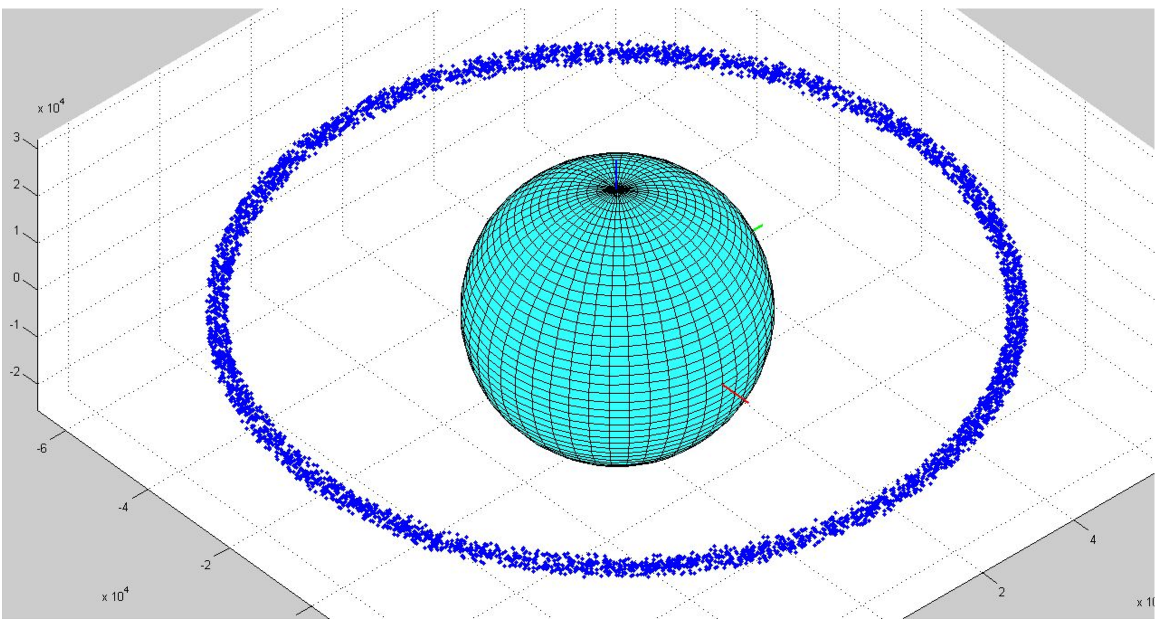

Figure 6. MATLAB generated model of Uranus and its Zeta ring

Using a script appended in Appendix D, The Zeta ring of Uranus was created. Essentially, the script takes the geometry of the ring, and populates the geometry with random points within a polar coordinate plane. From this, it will be possible to assign each of these points a random mass and the effective ring velocity in which the N-body problem will be analyzed. Figure 5 shows the model up close.

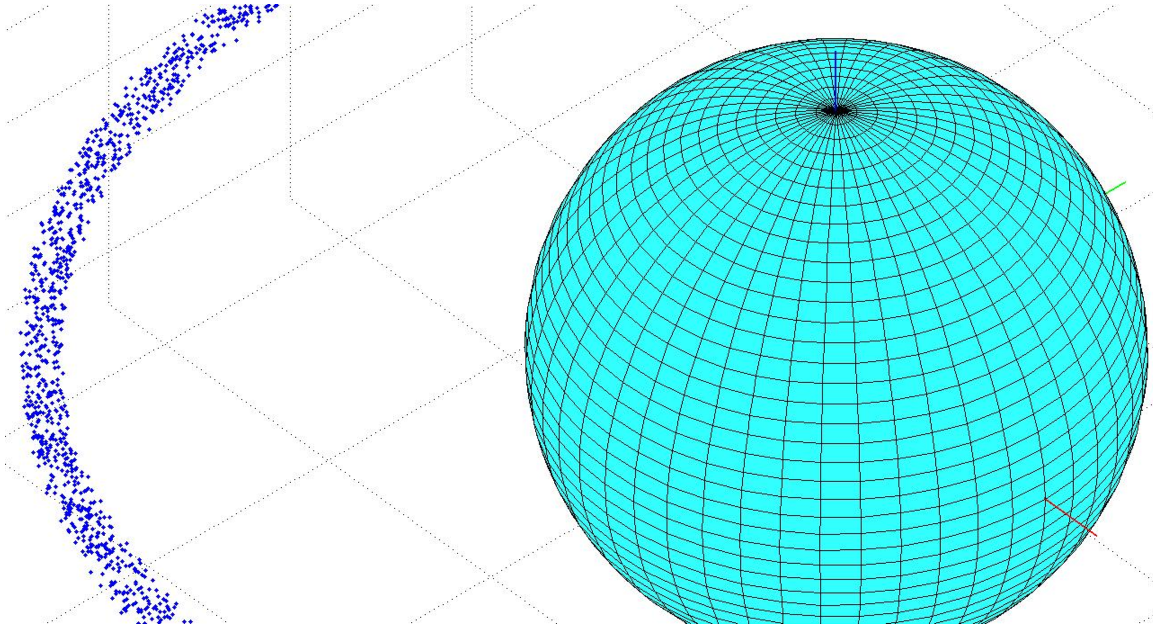

Figure 7. MATLAB generated model of Uranus zoomed in to see the individual points.

With this randomization, it will be possible to get random cases that could reflect what would possibly happen to a spacecraft in multiple flight scenarios to Uranus.

E. N-Body Problem

1. General Expression for N-Body Formulation

The N-Body Problem is the problem of calculating and predicting the motions of many bodies under mutual gravitational interaction. While analytically unsolvable, numerical integration approximations can determine the varying gravitational force acting on a given body, with respect to other masses of a dynamic system, to a high degree of accuracy. From this, it will be possible to track the trajectory of the given bodies with respect to one another. The figure below depicts an N-body system as presented by Bate, Muller, and White's Fundamentals of Astrodynamics.

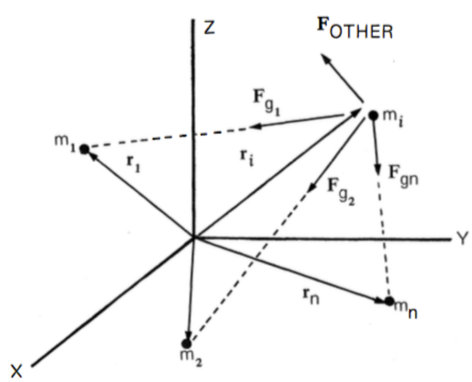

Figure 8. N-Body System: The term, F_{OTHER}, is the sum of all perturbing forces on object m_i, such as radiation pressure or atmospheric drag, and is assumed to be infinitesimally small and ignored in this analysis.

As with the 2 and 3-body problems, the N-body problem begins with the force vector, $\vec{\boldsymbol{F}}$ for a single object, defined by Newton's law of universal gravitation.

$$\vec{\boldsymbol{F}}_i = -\frac{Gm_i m_j}{\left|r_{ij}\right|^3}\vec{\boldsymbol{r}}_{ij} \tag{6.4}$$

Where G is the gravitational constant, m is the mass of an object, and r is the radial distance between objects i and j. The subscript i represents the body in question and subscript j represents the n^{th} body in the system. The vector $\vec{\boldsymbol{r}}_{ij}$, the distance between the initial and n^{th} body, is defined by the following:

$$\vec{\boldsymbol{r}}_{ij} = \vec{\boldsymbol{r}}_i - \vec{\boldsymbol{r}}_j \tag{6.5}$$

The sum of the forces on mass m_i due to N bodies becomes:

$$\vec{\boldsymbol{F}}_i = -Gm_i \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{m_j}{\left|r_{ij}\right|^3}\vec{\boldsymbol{r}}_{ij} \tag{6.6}$$

It is important to formulate the force vector as a sum, as seen in Equation (6.6), because it generalizes and simplifies the equations of motion for any number of N-bodies. The number of equations and terms become large as the quantity of bodies increase. Looking at Equation (6.6), the mass of m_i is constant and appears on both sides of the equality, so it can be divided out to further simplify the equations of motion for N-bodies to,

$$\frac{d^2\vec{\boldsymbol{r}}_i}{dt^2} = -G \sum_{\substack{j=1 \\ i\neq j}}^{N} \frac{m_j}{\left|r_{ij}\right|^3}\vec{\boldsymbol{r}}_{ij} \tag{6.7}$$

The equations of motion for the entire system are represented by the combined formulas derived by Equation (for bodies $i = 1, 2, \ldots, N$. From this general expression, an algorithm using N initial state vectors can be written to formulate N equations of motion. Then a solution to the N-body problem is found by applying one of many numerical integrators to these 2$^{\text{nd}}$ order equations over many iterations. For the sake of optimization and simplicity, the scope of the next part of this paper will be limited to 7 bodies conducted via MATLAB. The code will be provided in the next update. Appendix E provides a sample of the N-Body summations.

F. Placement Of The Particles

Using Matlab, the following vectors were produced showing the initial and final positions and velocities of each of seven particles that will be analyzed in a future paper. The particles are placed with respect to a Uranus fixed Z-Y frame. Each vector represents the initial and final conditions of the seven particles to be modeled for the N-Body solution, which will be scripted in the next part of this paper. Using a Matlab script, the positions and velocities of the seven particles were tracked for an allotted 1.4 hours; this is the time it takes for the spacecraft to dive between Uranus and it's Zeta ring. The final position and velocity were calculated from this. The script took the initial position inputs and from that calculated effective initial velocities as well as final position and velocity for the allotted time. With this, the N-Body problem is established and work toward determining a valid solution can now begin.

	Initial Position			Initial Velocity			Final Position			Final Velocity		
1	0	-41837	0	0	0	11.767	0	-6386.2	41343	0	11.63	1.795
2	0	-39789	-12928	0	-3.6361	11.191	0	-18849	37347	0	10.506	5.301
3	0	-33847	-24591	0	-6.9164	9.5196	0	-29468	29694	0	8.3536	8.288
4	0	-24591	-33847	0	-9.5196	6.9164	0	-37201	19134	0	5.3836	10.464
5	0	-12928	-39789	0	-11.191	3.6361	0	-41293	6702.2	0	1.8866	11.615
6	0	0	-41837	0	-11.767	0	0	-41343	-6386.2	0	-1.795	11.63
7	0	12928	-39789	0	-11.191	-3.6361	0	-37347	-18849	0	-5.301	10.506

Table 5. Initial and final position and velocity vectors for the seven ring particles.

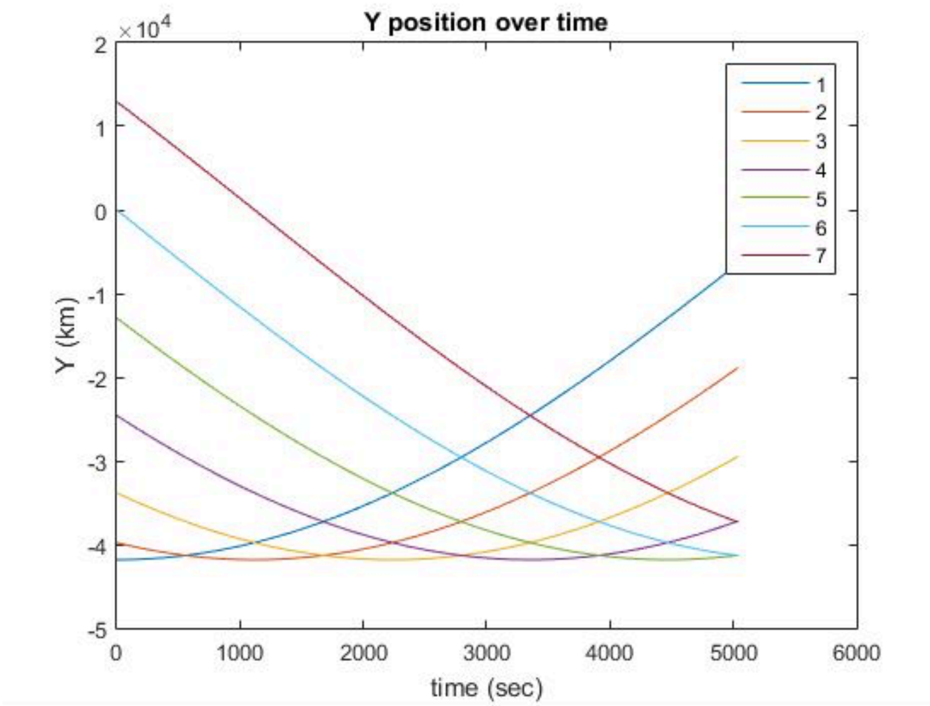

Figure 9. Y position of particles with respect to time.

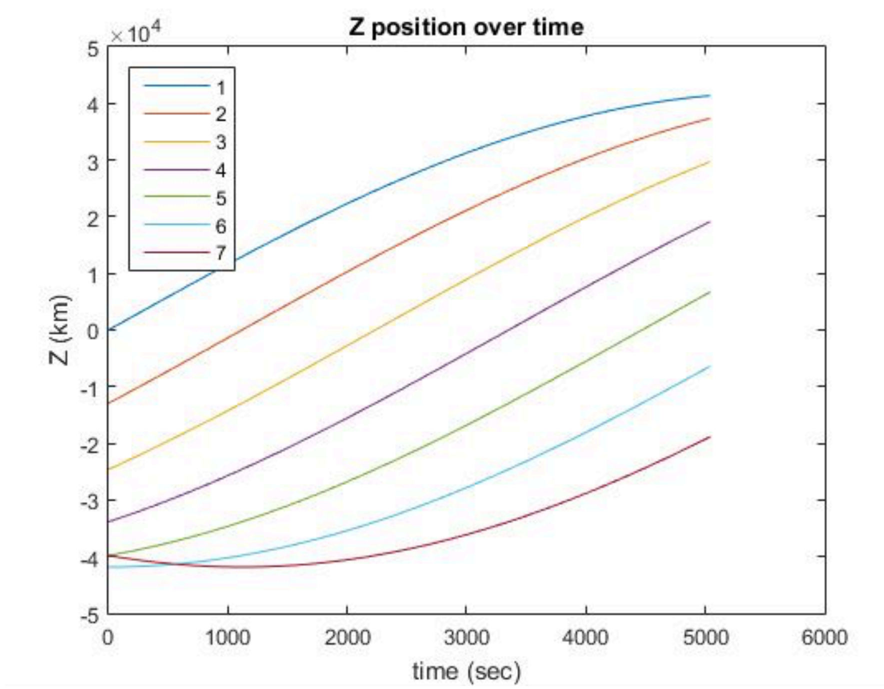

Figure 10. Z position of particles with respect to time.

G. N-Body Solver Overview

Because the ultimate objective of this research is to understand the orbital effects that inserting a spacecraft into lower Uranian orbit, between Uranus and Zeta, certain cases will be looked into in order to outline and benchmark these effects. It must be understood that there is no benchmark comparison for this analysis neither theoretically nor otherwise. As mentioned once previously, Uranus has not been a focus of study for scientists and thus this research is stand alone in what it may mean to perform an orbital insertion of a spacecraft into lower Oranocentric orbit. The effect of the rings particles and the perturbations that will occur on the satellite will be scrutinized.

The N-body analysis will be conducted using MATLAB. Said analysis conducted in this research is an adaptation based on A. Ahmed and L. Cohen's research, "A Numerical Integration Scheme for the N-Body Gravitational Problem". The numerical model and algorithm presented in their research provide a strong basis in providing realistic results for observing orbital perturbations due to injection.

The numerical integration begins with the second order differential equation where a solution is needed for the N bodies.

$$\frac{d^2 r_i}{dt^2} = -\sum_{\substack{j=1 \\ i\neq j}}^{N} \frac{(m_j r_{ij})}{|r_{ij}|^3} \quad \text{where } i = 1, 2, \dots N$$

Because integrating the equations of motion will prove to be tedious and difficult to achieve by hand, an ordinary differential equation solver will be designed in MATLAB. This paper will achieve the integration scheme through the design of a Runge-Kutta-Felberg 45 propagator and is compared to MATLAB's ODE23 propagator. Before any of that can be achieved, though, it must first be theorized and understood how the rings and spacecraft will have an effect on one another. In order to do this, the cases alluded previously will be conducted in the following order. First, a spacecraft of comparable mass to Voyager 2 will be placed in circular orbit between Uranus and Zeta. Second, that same satellite will conduct a flyby maneuver between the planet and its ring. Third a particle with the Earth's mass will be inserted in circular orbit between Uranus and Zeta; this particle will be referred to as the earth particle for the remainder of this paper. Fourth and finally, the earth particle will conduct the same flyby maneuver that Voyager 2 conducts in the previous case.

This paper theorizes that because the total mass of Zeta is significantly larger than that of Voyager 2, the rings will not perturb the satellite significantly. If perturbation of the spacecraft is initiated, it may be small enough to be considered negligible for the purposes of a real mission. The purpose of placing the earth particle and conducting a flyby maneuver between Uranus and Zeta will provide an idea of how large a mass would be necessary to perturb the ring system. With that understanding, the mass of Earth is approximately 5.972e24kg. If the N-body solution is integrated properly, the dynamics of the Uranus-Zeta system would be thrown into complete chaos.

Because the earth particle will undoubtedly disturb the dynamics of the ring system, we can use this information as a check to verify that the code produced is replicating the dynamics properly. Thus this extreme, and improbable case, will serve as a boundary condition and verification.

To prelude the investigation, figure x shows the simplified ring system that this paper investigates before the introduction of a satellite. Once Voyager 2 and the earth particle are introduced, there will be perturbations. Voyager 2 will most probably show no perturbations on the ring system because of its mass but the earth particle will surely throw the entire system into complete chaos.

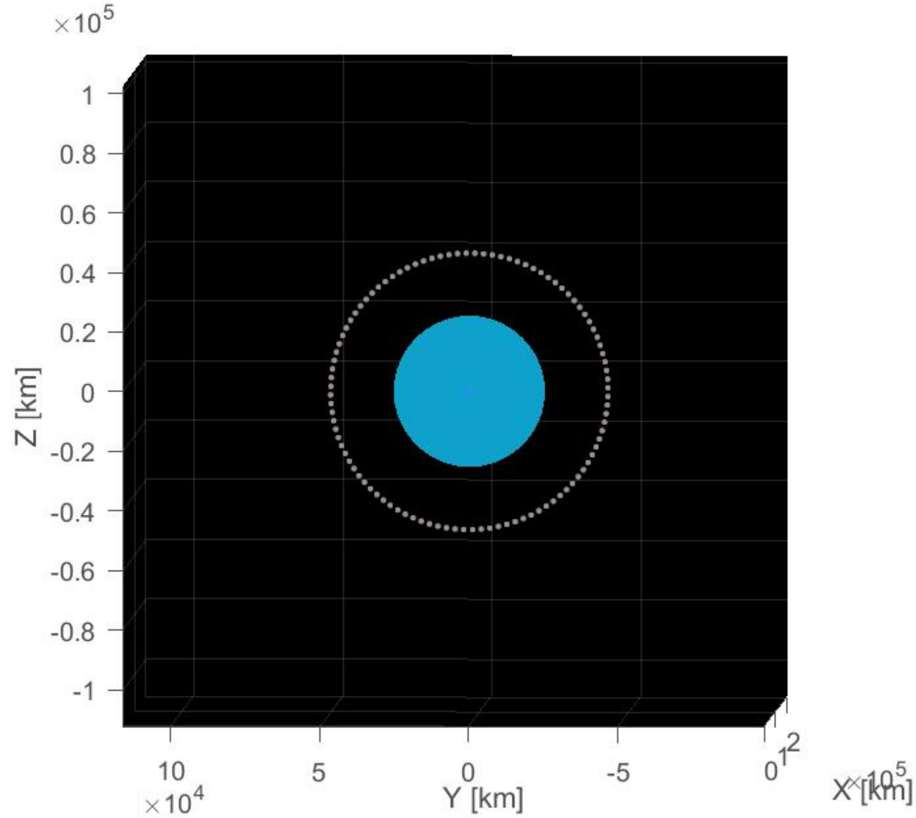

Figure 10. The figure shows the simplified Uranian ring system that is modeled in this paper with only 100 bodies in the lower ring, Zeta.

The significance of this data is to establish the system at equilibrium. Once new bodies are introduced, the system will be disturbed. As previously stated, depending on the mass of the object the magnitude of the disturbance will vary. It is the hope of this paper to verify that injecting a spacecraft with mass equivocal to Voyager 2 will show perturbations.

H. Initial Model Implementation

The following simulations are conducted with approximately 100 bodies over a course of fifteen hours. Although this may seem like an unrealistic model, computing time and computing power play a significant limiting factor. This model assumes that the total ring mass is approximately 7e16 kg. It is important to note that the ring with the model attempts to replicate that of Uranus as closely as possible. With that understood, the limitations presented do not show a real Uranian model, but a simplified model. Ideally, the research would conduct an analysis with the ring system consisting of over one million bodies over the course of many years. The code presented is capable of preforming this analysis with the proper computing machine.

I. Propagator

When comparing the results from MATLAB's ODE23 solver and the RKF45 Solver, we receive similar plots. The results did show some difference between propagators.

The results of the simulation will be discussed further in the next section but it is important to show a comparison between propagators to show the accuracy of the research. This investigation, as mentioned, uses both MATLAB's ODE23 and RKF45 propagators. The reason for this is to compare propagators for any future study conducted in research to Uranus. The preferred solver would be the generic RKF45 because it is generally accepted that the RKF45 produces a more accurate result than ODE23 because it is a higher order differential equation solver; it is also suspected that the RKF45 will be less computationally costly than the ODE23. Below is a comparison between the positional displacements in X, Y, and Z.

Other solvers, such as the Runge-Kutta-Felberg-86, could potentially provide the most optimal results but could also potentially be more computationally costly; the RKF 86 is a strong solver for analyses conducted over long periods of time and that is not the case for our analysis here.

Table 6: Comparison between the ODE 23 and RKF45 solvers for the polar orbit of Voyager 2

	X-displacement (km)	Y-displacement (km)	Z-displacement (km)
ODE23	0.0003348	0.0000887	0.0000223
RKF45	0.0003668	0.0002102	0.0000231

Table 7: Comparison between the ODE 23 and RKF45 solvers for the flyby maneuver of Voyager 2

	X-displacement (km)	Y-displacement (km)	Z-displacement (km)
ODE23	0.0005768	0.0000215	0.0001965
RKF45	0.0005893	0.0000088	0.0001949

Table 8: Comparison between the ODE 23 and RKF45 solvers for the polar orbit of the earth particle

	X-displacement (km)	Y-displacement (km)	Z-displacement (km)
ODE23	0.0083	0.0133	0.0003
RKF45	0.0005474	0.0001541	0.0000226

Table 9: Comparison between the ODE 23 and RKF45 solvers for the flyby maneuver of the earth particle

	X-displacement (km)	Y-displacement (km)	Z-displacement (km)
ODE23	0.001	0.0059	0.0021
RKF45	0.001	0.0059	0.0021

1. Solver Mathematical Explanation and Representation of the Runge-Kutta-Felberg-45

The Runge-Kutta-Felberg 45 is a numerical solver that approximates ordinary differential equations by solving them using fourth order and fifth order methods. It utilizes a predictor-corrector algorithm in which the solver will try to predict and correct the solution to the system of ordinary differential equations in question by solving them twice. By using a step size, h and h/2, the solver computes values utilizing both step sizes and compares their results. This produces two approximations per solution. If the approximations are in close agreement as dictated by an acceptable error then the solution is accepted. If there is disagreement in the solution, the step size will decrease and recalculate. Alternatively, if the solution is accepted, the step size increases and the solver will continue iterating. Each step requires a solution for each of these following values:

$$k_1 = hf(t_k, y_k) \tag{6.8}$$
$$k_2 = hf(t_k + \left(\frac{h}{4}\right), y_k + \left(\frac{h}{4}\right)k_1) \tag{6.9}$$
$$k_3 = hf(t_k + \left(\frac{h}{8}\right), y_k + \left(\frac{3}{32}\right)k_1 + \left(\frac{9}{32}\right)k_2) \tag{6.10}$$
$$k_4 = hf(t_k + \left(\frac{12h}{13}\right), y_k + \left(\frac{1932}{2197}\right)k_1 - \left(\frac{7200}{2197}\right)k_2 + \left(\frac{7296}{2197}\right)k_3) \tag{6.11}$$
$$k_5 = hf(t_k + h, y_k + \left(\frac{439}{216}\right)k_1 - 8k_2 + \left(\frac{3680}{513}\right)k_3 - \left(\frac{845}{4104}\right)k_4) \tag{6.12}$$
$$k_6 = hf\left(t_k + \left(\frac{h}{2}\right), y_k - \left(\frac{8}{27}\right)k_1 + 2k_2 - \left(\frac{3544}{2565}\right)k_3 + \left(\frac{1859}{4104}\right)k_4 + \left(\frac{11}{40}\right)k_5\right) \tag{6.13}$$

Each k value is written as a function as the previous *k* and step size of *h* or *h/2*.

The approximation for the solution is then made using Runge-Kutta method of order 4 and of order 5 respectively as follows:

$$y_{k+1} = y_k + \left(\frac{25}{216}\right)k_1 + \left(\frac{1408}{2565}\right)k_3 + \left(\frac{2197}{4101}\right)k_4 - \left(\frac{1}{5}\right)k_5 \tag{6.14}$$
$$z_{k+1} = y_k + \left(\frac{16}{135}\right)k_1 + \left(\frac{6656}{12825}\right)k_3 + \left(\frac{28561}{56430}\right)k_4 - \left(\frac{9}{50}\right)k_5 + \left(\frac{2}{55}\right)k_6 \tag{6.15}$$

Then, finally, the optimal step size sh would be determined by multiplying a scalar *s* to *h*. The value *s* is found as follows:

$$s = \left(\frac{h}{(2|z_{k+1} - y_{k+1})}\right)^{\frac{1}{4}} \tag{6.16}$$

VI. Solutions

A. Polar Orbit – Voyager 2

1. Analysis

The introduction of Voyager 2 into Oranocentric polar orbit shows that there is very little perturbation on the spacecraft. The magnitude of the perturbation is approximately 42.33909423 cm. The plots below show the displacements of the rings with respect to the spacecraft.

2. Discussion

The masses in the system are almost all in equilibrium. The rings, as shown below, do not show displacement. Likewise, the spacecraft itself experiences minimal perturbations. This is in line with our predictions. Voyager 2 is too small of a body to have an effect on the rings. As such, the masses in the system perturb the spacecraft but only minimally. This perturbation can be considered negligible.

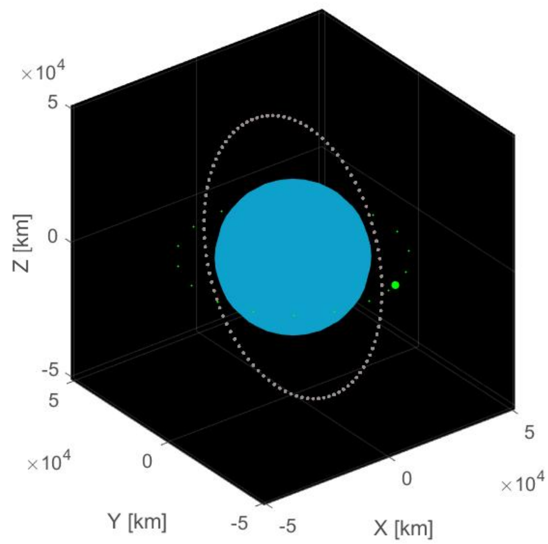

Figure 11. Visual representation of the Uranian system as Voyager 2 orbits between Zeta and Uranus in polar orbit.

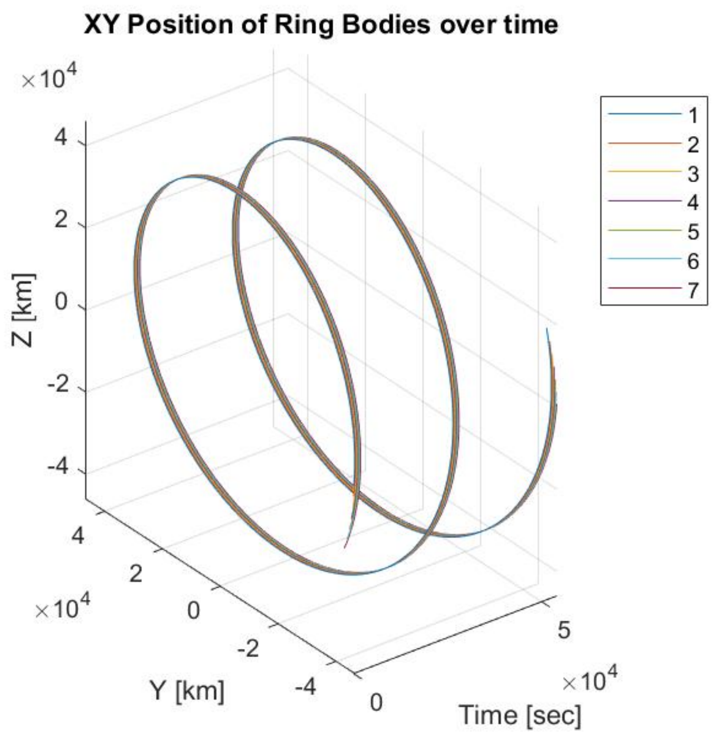

Figure 12. 3 dimensional plot showing a sample of 7 particles in Zeta as Voyager 2 is in polar orbit.

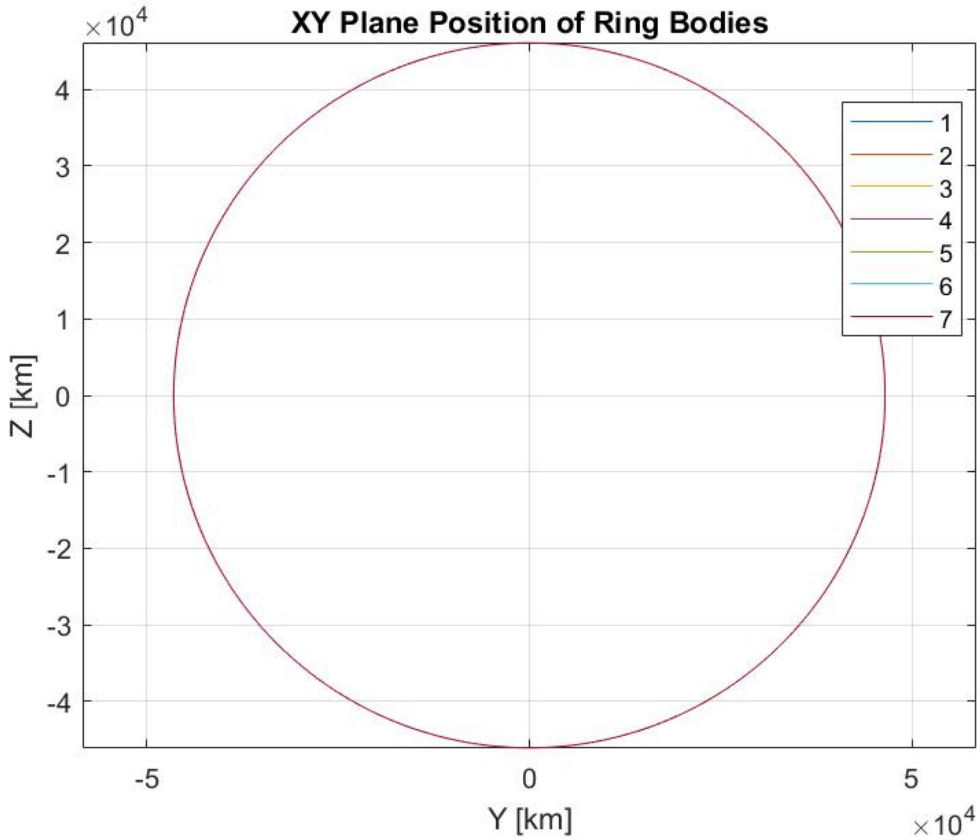

Figure 13. 2 dimensional plot showing a sample of 7 particles in Zeta as Voyager 2 is in polar orbit.

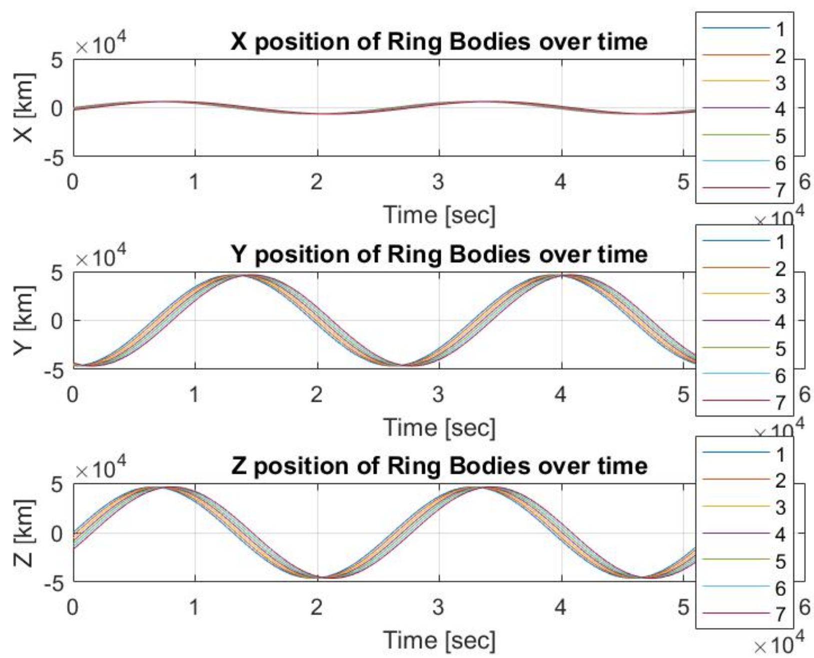

Figure 14. Visual representation of the displacements of the particles in 1 dimension over time.

B. Polar Orbit – Earth Particle

1. Analysis

Earlier on in this paper, it had been predicted that the earth particle would throw the entirety of the Uranian-ring system into disarray due to its mass. Because the mass of the earth particle is so large, it was also predicted that the effect that the rings had on it would be comparatively infinitesimal. As dictated by the solver, the magnitude of the earth particle's displacement is approximately 56.91259351 cm.

2. Discussion

These results set a necessary precedent for this solver. Because we cannot compute results over a significant amount of time with the necessary number of bodies to create a more realistic analysis, this analysis was meant to immediately disturb the system. Previously, it had been mentioned that the analyses regarding the earth particle were meant to be a verification. This analysis does in fact prove our suspicions and proves the functionality of this code.

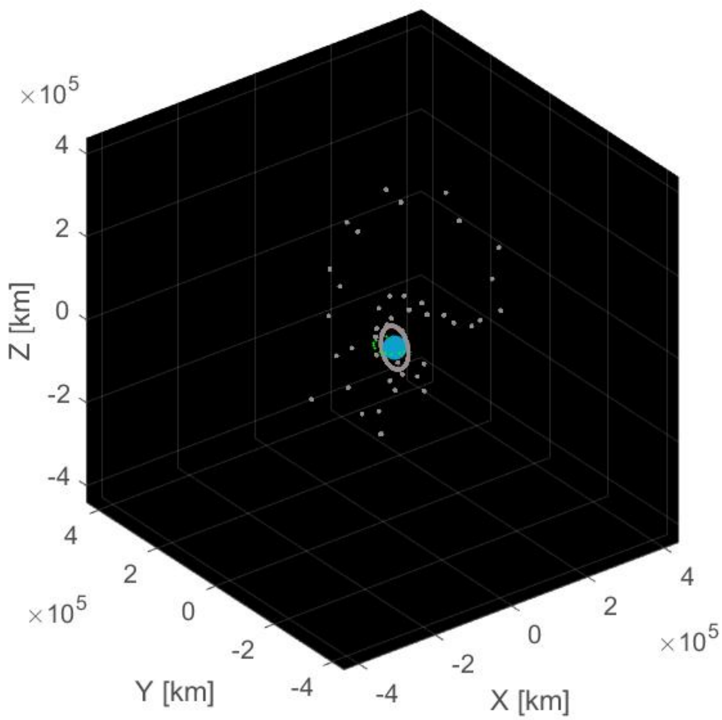

Figure 15. Visual representation of the Uranian system as the earth particles orbits its poles.

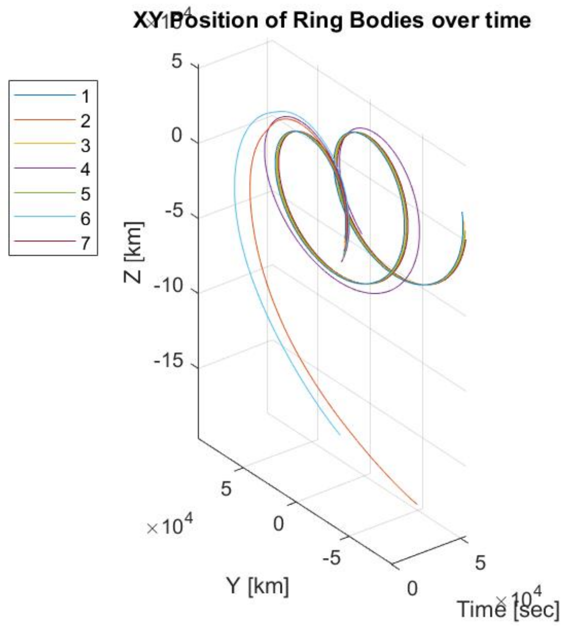

Figure 16. The figure shows a 3 dimensional representation of the earth particles polar orbit around Uranus. Each color represents the changing position of each sample ring particle as the earth particle orbits Uranus.

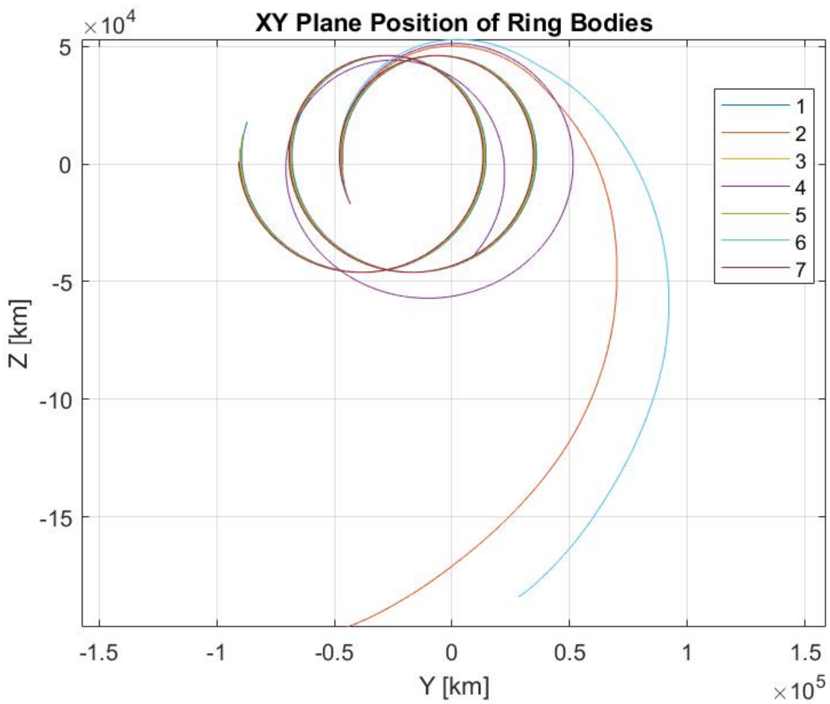

Figure 17. A 2 dimensional sample of seven particles in Zeta as a earth particle orbits Uranus' poles.

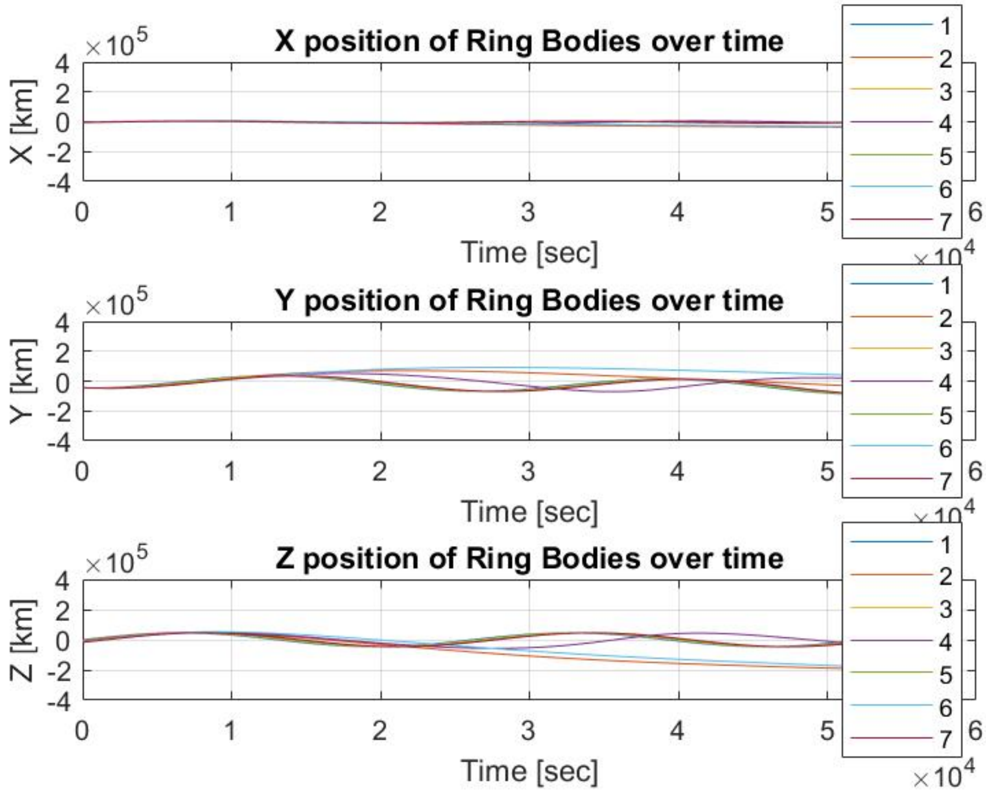

Figure 18. Visual representation of the displacements of the particles in 1 dimension over time.

C. Flyby Maneuver – Voyager 2

1. Analysis

As one may suspect, conducting a flyby maneuver produces less perturbation than a consistent plethora of orbits would on the spacecraft. This is due to the obvious constant gravitational effects on the spacecraft as it orbits the planet. In the case presented here, because we are only orbiting the planet over a course of 15 hours, this cannot be seen and is thus not exemplified. The flyby maneuver actually produces a larger perturbation on the spacecraft than the polar orbit does in this short time period; the effective perturbation yielded upon Voyager 2 in this case is approximately 62.07559424 cm.

2. Discussion

Theoretically, if the spacecraft is in orbit over a longer period of time, the spacecraft should experience increased levels of perturbation. The bodies affecting the spacecraft would consistently "pull" on it. In the case of the flyby, as presented here, it is only pulled on momentarily as the spacecraft is passing through the ring system.

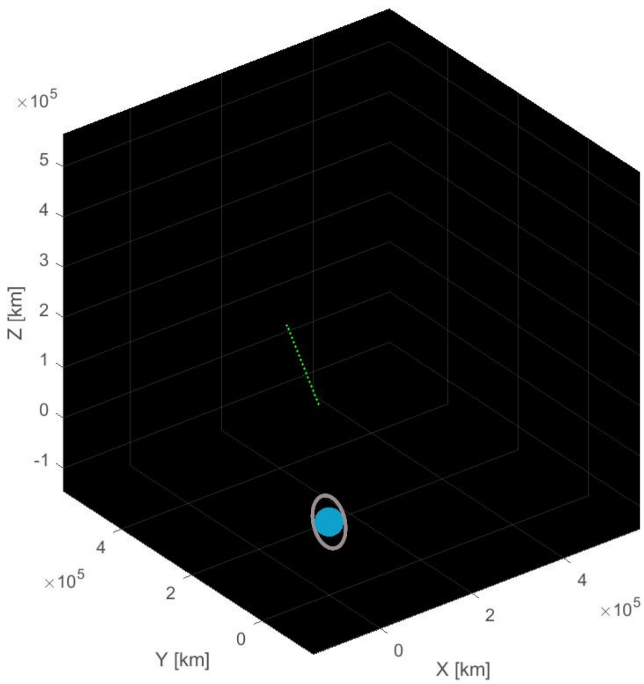

Figure 19. The visualization shows the spacecraft as it flies through the ring.

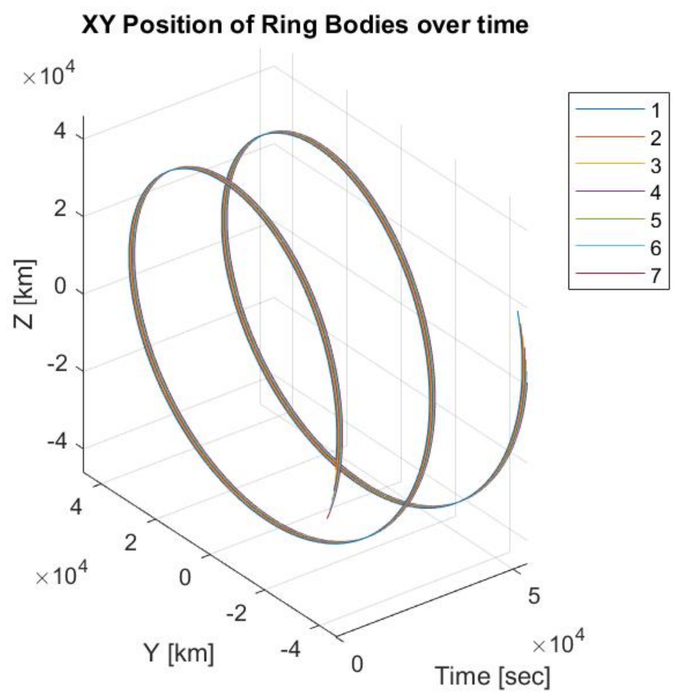

Figure 20. The figure above looks almost identical to figure X. This is due to the very small perturbations that the spacecraft has on the rings and vise versa.

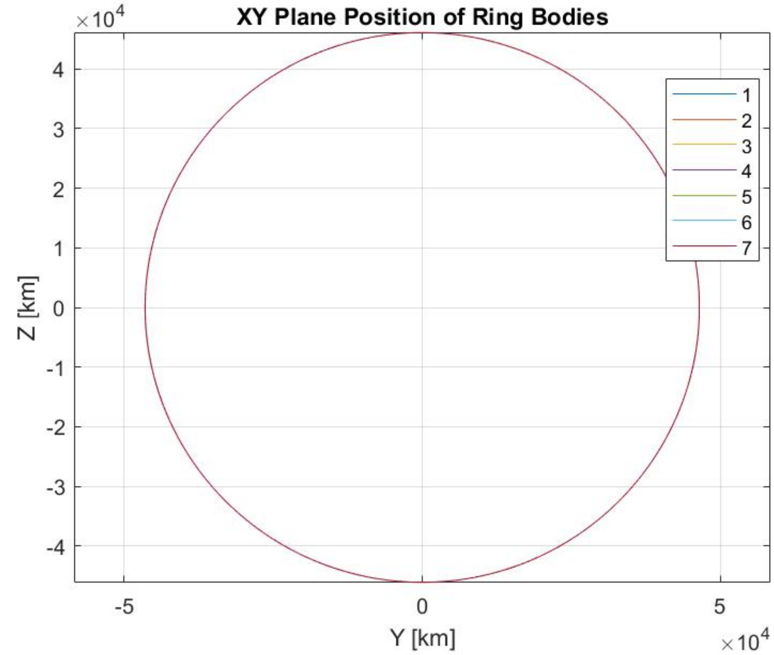

Figure 21. Similar to the previous figure, the figures between the flyby and polar orbit are indistinguishable because of the quantity at which the spacecraft is being perturbed.

D. Flyby Maneuver – Earth Particle

1. Analysis

The mass of the earth particle, as insinuated in its name, is enormous. When the earth particle flies between Uranus and Zeta, it throws the system into chaos as it does when it is in polar orbit. But, it is seen in this case that the particle perturbs the system much less than in polar orbit. The earth particle, though, is perturbed considerably more than with the polar orbit case. It is perturbed by 6.341924 m. As mentioned in the analysis of the Voyager 2 flyby maneuver, the same stands true here.

2. Discussion

Something that requires further understanding is why such a large mass is perturbed so much more considerably than its smaller counterpart in the previous case. It is expected that there is some error in this, perhaps in the initialization of the RKF45 solver, but the results are well within believable physics.

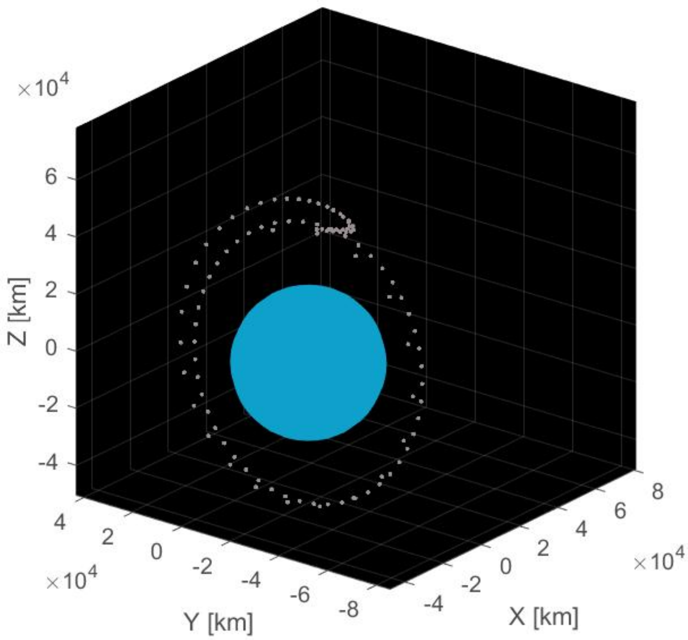

Figure 22. Visual representation of the Uranian system as the earth particle passes between Zeta and the planet. The flyby also throws the system into complete disarray.

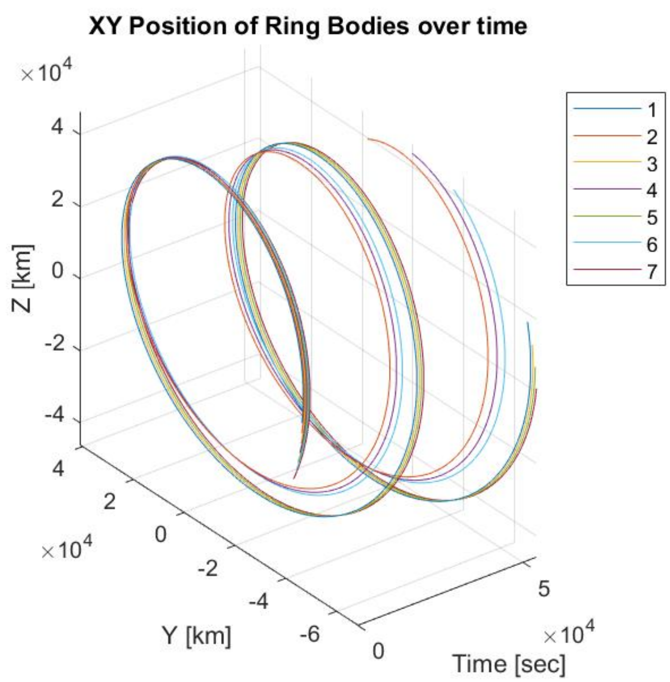

Figure 23. A 3 dimensional diagram of the ring system is perturbed as the earth particle flies between Uranus and Zeta.

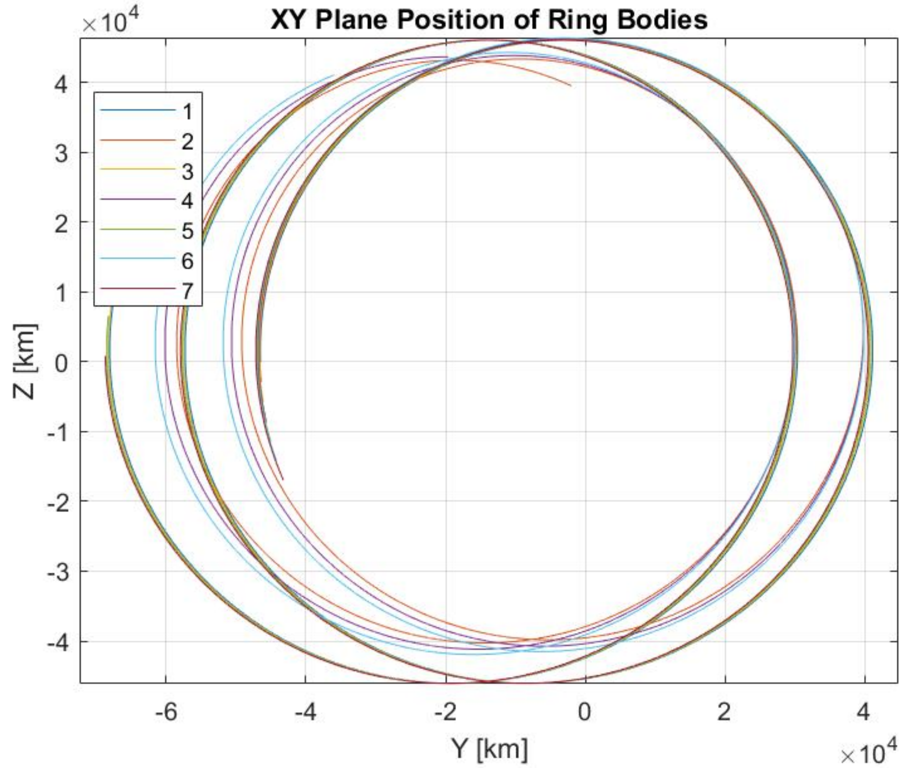

Figure 24. A 3 dimensional diagram of the ring system is perturbed as the earth particle flies between Uranus and Zeta. Perturbation is noticeably smaller on the rings here.

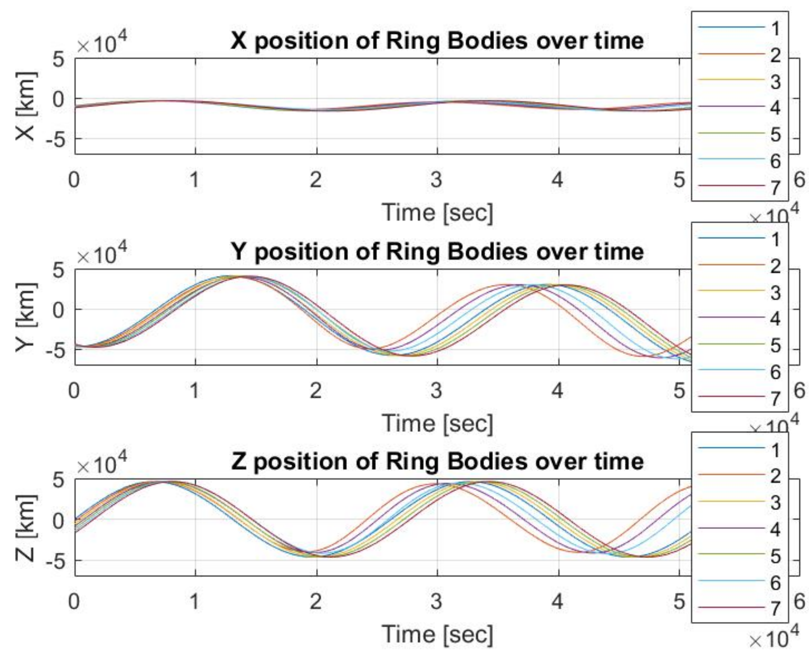

Figure 25. The one dimensional position changes for the earth particles flyby maneuver are shown.

VII. Conclusion

From the research conducted in this investigation, a mission to the lower ring of Uranus seems to be very possible. Whether a flyby maneuver is to be conducted or a polar orbit between the planet and its lower ring, the research presented platforms the possibility of a future mission. With much more powerful computing hardware, a more accurate representation of this analysis could be analyzed and a better understanding of spacecraft – ring interactions could be achieved. This research hopes to set the grounds and interest in possible missions to Uranus with the understanding that more research can be, and should be, conducted with a more accurate model including all thirteen rings, the moons, and a more accurate transit between the earth and Uranus.

VIII. Acknowledgments

I would like to thank San Jose State University for allowing me to conduct my research under its guise and Professor Jeanine Hunter for guiding me and providing support as I conducted my research. I would also like to thank my colleagues, Theodore Hendrix and Andrew Torricelli for assisting me with writing my MATLAB scripts. I would like to thank my father and mother for their guidance and support, which has culminated in my achievements and ability to preform this research. Finally, I would like to thank the late Dr. Fawzi Karajeh for the inspiration, motivation, and unequivocal support in my years leading up to the conclusion of this article.

References

Ahmad, A., & Cohen, L. (1973). A numerical integration scheme for the N-body gravitational problem. Journal of Computational Physics, 12(3), 389-402.

Bate, R. R., Mueller, D. D., & White, J. E. (1971). Fundamentals of astrodynamics. Courier Corporation. Chicago

Chaple, G. F., *Greenwood Guides to the Universe: Outer Planets*, Santa Barbra, Ca: ABC-CLIO, 2009.

Lunine, J. I., "The Atmospheres of Uranus and Neptune," *Annual Review of Astronomy and Astrophysics*, vol. 31, pp. 217–263.

Pater, I. D., "New Dust Belts of Uranus: One Ring, Two Ring, Red Ring, Blue Ring," *Science*, vol. 312, Jul. 2006, pp. 92–94.

Pater, I. de., and Lissauer, J. J., *Planetary sciences*, Cambridge: Cambridge Univ. Press, 2011.

N. A. S. A., "Uranus Lithograph" Available: https://www.nasa.gov/sites/default/files/files/Uranus_Lithograph.pdf.

APPENDIX A.

A. Jupiter Hohmann.m

```
%clear all; close all; clc;
%% Student: Zaid Karajeh
% Master's Advisor: Jeanine Hunter
%1) Hohmann from Earth to Jupiter (for flyby)
% Define Earth (r_p) and Jupiter (r_a) and mu_earth and
mu_sun
r_a = 816.62e6 % Km Jupiter
r_p = 147.09e6 % Km Earth
```

```
r_p_E = (6371 + 500) % Distance at Earth
r_p_J = (71492+200000) %Distance at Jupiter
mu_earth = 0.39869e6 % Km^3/s^2
mu_sun = 132712e6 %Km^3/s^2

mu_jupiter = 126.687e6 %Km^3/s^2
%period of jupiter
per_jupiter = sqrt((4*pi^2*r_a^3)/(mu_sun))
%period of earth
per_earth = sqrt((4*pi^2*r_p^3)/(mu_sun))
% Calculate Semi Major Axis of Transfer Ellipse, a
a=.5*(r_a + r_p) %Km
% Calculate Ecentricity of Transfer Ellipse, e
e = (r_a - r_p)/(r_a + r_p)
%Hyperbolic excess Velocity Earth wrt Q
E_V_Q = sqrt(mu_sun/r_p) * (sqrt((2*r_a)/(r_a+r_p))-1) % sy
%Burn from 500Km LEO to departure hyperbola
Del_V_1   =   sqrt((E_V_Q)^2   +   (2*mu_earth)/(6878))   -
sqrt(mu_earth/6878)
%Hyperbolic excess velocity jupiter wrt Q
J_V_Q  =  sqrt(mu_sun/r_a)  *  (sqrt(1-((2*r_p)/(r_a+r_p))))%
sy
%% Hyperbola parameters
%Aim radius at jupiter
Aim_Rad_Juptier=r_p_J*
(sqrt(1+((2*mu_jupiter)/(r_p_J*J_V_Q^2))))
%Aim radius at earth
Aim_Rad_Earth=r_p_E*
(sqrt(1+((2*mu_earth)/(r_p_E*E_V_Q^2))))
%Asymptote angle, beta at earth in degrees
beta_earth = acos(1/(1+((r_p_E * (E_V_Q^2))/(mu_earth))));
rad2deg(beta_earth)
%Asymptote angle, beta at Jupiter in degrees
beta_Jupiter=acos(1/(1+((r_p_J* (J_V_Q^2))/(mu_jupiter))));
rad2deg(beta_Jupiter)
%% Phasing
% Period (1) of transfer orbit to Jupiter from earth
peri_tr_1 = (1/2) * sqrt((4*pi^2*a^3)/(mu_sun))
%angular velocities
omega_j = (2*pi/per_jupiter) %angular velocity of jupiter
omega_e = (2*pi/per_earth) %angular velocity of uranus
%Theta_delta   for   target   (angle   jupiter   covers   during
transfer)
%period of transfer ellipse * angular velocity
theta_delta_jupiter = omega_j * peri_tr_1
rad2deg(theta_delta_jupiter)
%phase   angle   jupiter   must   be   at   when   transfer   begins
relative to earth

final_phase_j = pi - theta_delta_jupiter
rad2deg(final_phase_j)
total_phase = final_phase_j + final_phase_u
```

```
rad2deg(total_phase)
%% Determine Launch date
TA_earth = deg2rad(1.302377841813924e2) %on may 12 2017 in
rad (courtesy of JPL Horizons)
TA_jupiter = deg2rad(1.863005081850869e2) %on may 12 2017
in rad (courtesy of JPL Horizons)
TA_uranus = deg2rad(2.114707033271004e2)%on may 12 2017 in
rad (courtesy of JPL Horizons)
t=0 %time index
tstep = 60
tol= pi/128
C_PA_jupiter = abs(TA_jupiter - TA_earth) %phase angle
between 2 planets at defined moment in time
C_PA_uranus = abs (TA_uranus - TA_jupiter)%phase angle
between 2 planets at defined moment in time
while(1) %begin stepping forward in time
    if TA_earth >= 2*pi
        TA_earth = TA_earth + (tstep* omega_e)-2*pi;
    else
        TA_earth = TA_earth + (tstep* omega_e);
    end
    if TA_jupiter >= 2*pi
        TA_jupiter = TA_jupiter + (tstep* omega_j)-2*pi;
    else
        TA_jupiter = TA_jupiter + (tstep* omega_j);
    end
    if TA_uranus >= 2*pi
        TA_uranus = TA_uranus + (tstep* omega_u) - 2*pi;
    else
        TA_uranus = TA_uranus + (tstep* omega_u);
    end
    if (TA_jupiter - TA_earth < 0)
    C_PA_jupiter = 2*pi+TA_jupiter - TA_earth;
    else
      C_PA_jupiter =  TA_jupiter - TA_earth;
    end
    if (TA_uranus - TA_jupiter < 0)
        C_PA_uranus = 2*pi + TA_uranus - TA_jupiter;
    else

       C_PA_uranus = TA_uranus - TA_jupiter;
    end
    t=t+1;
    if mod(t*tstep,3.154e8) == 0
       years =  t*tstep/3.154e7;
       fprintf('%.3f Years\n',years)
    end
    if ((C_PA_jupiter - tol <= final_phase_j &&
C_PA_jupiter + tol >= final_phase_j) && (C_PA_uranus - tol
<= final_phase_u && C_PA_uranus + tol >= final_phase_u))
        break
    end
```

```
    end
    wait_time = t*tstep
```

B. Uranus Hohmann.m

```
%clear all; close all; clc;
%% Student: Zaid Karajeh
% Master's Advisor: Jeanine Hunter
%1) Hohmann from Jupiter to Uranus (Arrival Planet)
% Define Jupiter (r_p) and Uranus (r_a) and mu_uranus and mu_sun
r_a = 2741.30e6 % Km Uranus
r_p = 816.62e6 % Km Jupiter
mu_uranus = 5.7940e6 % Km^3/s^2
mu_sun = 132712e6 %Km^3/s^2
mu_jupiter = 126.687e6 %Km^3/s^2
%period of jupiter
per_jupiter = sqrt((4*pi^2*r_p^3)/(mu_sun))
%period of uranus
per_uranus = sqrt((4*pi^2*r_a^3)/(mu_sun))
% Calculate Semi Major Axis of Transfer Ellipse, a
a=.5*(r_a + r_p) %Km
% Calculate Ecentricity of Transfer Ellipse, e
e = (r_a - r_p)/(r_a + r_p)
%Hyperbolic excess Velocity Jupiter wrt Q
J_V_Q = sqrt(mu_sun/r_p) * (sqrt((2*r_a)/(r_a+r_p))-1) % sy
%Burn from 200000Km LJO to departure hyperbola

Del_V_1  =  sqrt((J_V_Q)^2  +  (2*mu_jupiter)/(71492+200000))  -
sqrt(mu_jupiter/(71492+200000))
%Hyperbolic excess velocity Uranus wrt Q
U_V_Q = sqrt(mu_sun/r_a) * (sqrt(1-((2*r_p)/(r_a+r_p)))) % sy
%% Phasing
% Period (2) of transfer orbit to Uranus from Jupiter
peri_tr_2 = (1/2) * sqrt((4*pi^2*a^3)/(mu_sun))
%angular velocities
omega_j = (2*pi/per_jupiter) %angular velocity of jupiter
omega_u = (2*pi/per_uranus) %angular velocity of uranus
%Theta_delta for target (angle jupiter covers during transfer)
%period of transfer ellipse * angular velocity
theta_delta_uranus = omega_u * peri_tr_2
rad2deg(theta_delta_uranus)
%phase angle jupiter must be at when transfer begins relative to
earth
final_phase_u = pi - theta_delta_uranus
rad2deg(final_phase_u)
```

C. Delta V Calculator

```
%% Delta-V Calculator
% Orbit Altitude Definitions altitude = [ r_p_E, r_p_J, r_p_J,
r_p_U]; % Planet Velocities v_body_e = sqrt(mu_sun./147.09e6);
```

```
v_body_j = sqrt(mu_sun./816.62e6); v_body_u =
sqrt(mu_sun./2741.30e6);
% Trasnfer Ellipse Velocities v_peri_e = sqrt(2*mu_sun./147.09e6-
mu_sun./a);
v_peri_j = sqrt(2*mu_sun./816.62e6-mu_sun./a);
v_apo_j = sqrt(2*mu_sun./816.62e6-mu_sun./a);
v_apo_u = sqrt(2*mu_sun./2741.30e6-mu_sun./a);
%Hyperbolic excess velocity v_infinity = [v_peri_e - v_body_e,
v_body_j - v_apo_j, v_peri_j - v_body_j,...
 v_body_u - v_apo_u];
% Hyperbolic Eccentricity hyperbolic_e = 1 +
altitude.*v_infinity.^2./[mu_earth, mu_jupiter, mu_jupiter,
mu_uranus];
% Turning Angle del_turn = 2*asin(1./hyperbolic_e);
% Delta-V Calculations v_inf_departure =
[v_infinity(2)*cos(del_turn(2)) v_infinity(2)*sin(del_turn(2))];
delta_v = [sqrt(v_infinity(1)^2 + 2*mu(2)/altitude(1))-
sqrt(mu(2)/altitude(1))
norm([v_infinity(3)+v_inf_departure(1), 0-v_inf_departure(2)])
sqrt(mu(4)/altitude(4)) - sqrt(v_infinity(4)^2 +
2*mu(4)/altitude(4))];
Delta_v_total = sum(abs(delta_v));
row_names = {'Earth Departure','Jupiter Correction','Uranus
Arrival','Total'};
Dv_total = table(cat(1,delta_v,
Delta_v_total),'rownames',row_names,...
'variablenames',{'Delta_V'})
```

D. MATLAB script Generating Uranus and it's Zeta Ring

```
clc, clear all, close all
% Create a random set of coordinates in a circle.
% First define parameters that define the number of points and the
circle.
[radius, mu, Rp, Ra, soi, rgb] = orbital_constants('Uranus');
n = 50000;
Outter_R = 41378+radius;
Inner_r = 38000+radius;
x0 = 0; % Center of the circle in the x direction.
y0 = 0; % Center of the circle in the y direction.
z0 = 0;
% Now create the set of points.
% For a full circle, use 0 and 2*pi.
angle1 = 0;
angle2 = 2*pi;
z_thickness = 2500;
% For a sector, use partial angles.
% angle1 = pi/4;
% angle2 = 3*pi/4;
t = (angle2 - angle1) * rand(n,1) + angle1;
r = Outter_R*sqrt(rand(n,1));
x = x0 + r.*cos(t);
y = y0 + r.*sin(t);
```

```
a = (z0 - z_thickness/2);
b = (z0 + z_thickness/2);
z = a + (b-a).*rand(n,1);
data = cat(2,x,y,z);
indx = find(sqrt(data(:,1).^2 + data(:,2).^2 + data(:,3).^2) >=
Inner_r);
%indx = find(data(:,1) ~= 0);
% Now display our random set of points in a figure.
hold on
body_plot(radius, rgb,1)
plot3(data(indx,1), data(indx,2), data(indx,3), '.', 'MarkerSize',
5)
axis equal;
grid on;
```

E. N-Body Summations

Example of 4-body system, equations of motion $\left(r_{ij} = r_{ji}\right)$:

$$\frac{d^2\vec{\boldsymbol{r}}_1}{dt^2} = -G\left(\frac{m_2}{|r_{12}|^3}\,\vec{\boldsymbol{r}}_{12} + \frac{m_3}{|r_{13}|^3}\,\vec{\boldsymbol{r}}_{13} + \frac{m_4}{|r_{14}|^3}\,\vec{\boldsymbol{r}}_{14}\right)$$
$$\frac{d^2\vec{\boldsymbol{r}}_2}{dt^2} = -G\left(\frac{m_1}{|r_{21}|^3}\,\vec{\boldsymbol{r}}_{21} + \frac{m_3}{|r_{23}|^3}\,\vec{\boldsymbol{r}}_{23} + \frac{m_4}{|r_{24}|^3}\,\vec{\boldsymbol{r}}_{24}\right)$$
$$\frac{d^2\vec{\boldsymbol{r}}_3}{dt^2} = -G\left(\frac{m_1}{|r_{31}|^3}\,\vec{\boldsymbol{r}}_{31} + \frac{m_2}{|r_{32}|^3}\,\vec{\boldsymbol{r}}_{32} + \frac{m_4}{|r_{34}|^3}\,\vec{\boldsymbol{r}}_{34}\right)$$
$$\frac{d^2\vec{\boldsymbol{r}}_4}{dt^2} = -G\left(\frac{m_1}{|r_{41}|^3}\,\vec{\boldsymbol{r}}_{41} + \frac{m_2}{|r_{42}|^3}\,\vec{\boldsymbol{r}}_{42} + \frac{m_3}{|r_{43}|^3}\,\vec{\boldsymbol{r}}_{43}\right)$$

F. Sphere of Influences of the Moons of Uranus

Sphere of Influence:	km		R	Mb	MB
Uranus	51760998.27		2870970000	8.68E+25	1.99E+30
Cordelia	9.522779139		49,752	4.40E+16	8.68E+25
Ophelia	11.08597814		53,764	5.30E+16	8.68E+25
Bianca	15.21080145		59,165	9.20E+16	8.68E+25
Cressida	26.78675914		61,767	3.40E+17	8.68E+25
Desdemona	21.07000114		62,659	1.80E+17	8.68E+25
Juliet	34.22011496		64,630	5.60E+17	8.68E+25

Portia	54.56741923		66,097	1.70E+18	8.68E+25
Rosalind	26.81593829		69,927	2.50E+17	8.68E+25
Cupid	5.375107248		74,800	3.80E+15	8.68E+25
Belinda	33.3909256		75,255	3.60E+17	8.68E+25
Perdita	10.23029443		76,420	1.80E+16	8.68E+25
Puck	87.9123617		86,004	2.90E+18	8.68E+25
Mab	4.117349857		97,734	1.00E+15	8.68E+25
Miranda	461.3462302		129,390	6.59E+19	8.68E+25
Ariel	2281.235161		191,020	1.35E+21	8.68E+25
Umbriel	3002.716349		266,300	1.17E+21	8.68E+25
Titania	7637.146357		435,910	3.53E+21	8.68E+25
Oberon	9600.307475		583,520	3.01E+21	8.68E+25
Francisco	396.7736226		4,276,000	7.20E+15	8.68E+25
Caliban	2772.978246		7,231,000	2.50E+17	8.68E+25
Stephano	1161.042854		8,004,000	2.20E+16	8.68E+25
Trinculo	617.4775778		8,504,000	3.90E+15	8.68E+25
Sycorax	11731.66712		12,179,000	2.50E+18	8.68E+25
Margaret	1195.137094		14,345,000	5.50E+15	8.68E+25
Prospero	4049.052039		16,256,000	8.50E+16	8.68E+25
Setebos	4126.6239		17,418,000	7.50E+16	8.68E+25
Ferdinand	1728.60859		20,901,000	5.40E+15	8.68E+25